

THE ROLE OF INCOMES IN DISCRETE CHOICE MODELS

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This Dissertation is submitted by the Candidate
in Partial Fulfilment of the Requirements for the Degree of

MA Transport Economics

Submission by the Candidate does not imply
that its content or standard is endorsed by
the Examiners

Institute for Transport Studies

The University of Leeds

September 2014

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EXECUTIVE SUMMARY

In the last forty years academic research in discrete choice models with foundations in microeconomic principles has developed an attractive framework for the detection of income effects. Nevertheless, a common practice is to assume that the specification of incomes within models, when calculating demands or even welfare measurement, is innocuous. Large investments in transport sector nowadays entail aggregation of benefits across individuals; this latter leads to require sophisticated tools and deep analysis for accurate measures. In this context, a mis-specification of incomes in transport models may potentially distort outcomes and unintentionally conduct to non-optimal decisions.

The objectives of this dissertation come from the theoretical and empirical approaches. From the theoretical field, this study aims to examine the relevant literature of income effect in discrete choice models so as to provide a straightforward and illustrative document in this subject. The empirical objective pursues to detect the presence of income effects in the Peruvian case of study so as to contribute with some insights of the role of income for future transport investment appraisal in this country.

The case of study is applied for the Lima Metropolitan Area. Since the 90' the governmental policies in the transport sector has been oriented to lax some rules concerning to entry and exit, quality of service and pricing deregulation. Although these policies initially were promoted with the intention to overcome the excess of demand, then the extreme flexibility in the rules raised congestion problems, lack of incentives for investing in capital, reduction in travel safety, among other.

The central Peruvian government has resumed and undertaken some projects in the urban transport sector; in particular, projects in urban metro. The investment of these projects entails a considerable amount of resources, thus a profound analysis of the benefits need to be carefully done. The ignorance or the mis-specification of income effects into the appraisal may distort the welfare benefits of these projects. Thus the overall benefits may not outweigh the losses in the transport sector. For this reason, the empirical tests in the present study attempt to identify the presence of income effects and potentially implications in the welfare measurement

On the basis of discrete choice theory, the present study has tested several models by using the multinomial logit models. The dataset collected from an urban transport survey Lima Metropolitan in 2004 has enabled to carry several models. The first set of models has sought to detect the presence of income effects, thus by assuming an income tier it has been found the individuals surveyed in the study can be classified into two representative income-groups. An additional test, by following the Jara-Diaz and Videla (1989), shows that the marginal utility of income decreases with income and these are statistically different when are calculated among groups. As in the author's work, these results may induce to further research in this field for the case of income effects in developing countries.

By assuming the set of specifications in proposed in Batley and Ibanez (2013), several models have been carried out. It has found the models are sensitive to the specification form; indeed it can be seen that some models might yield substantial different results even when they come from the same dataset. In this sense, since the dataset was constructed from a revealed preference survey, it may be reasonable presume to find higher variations in the values of the attributes which in some extend may be reflected in the models with different specifications. A final testing group is regarding to the measure of logsum, for this purpose variations in the size of the time and cost variables have been introduced so as to replicate hypothetical scenarios that might been observed with the introduction of the metro system. Although most of the models are consistent with the expected pattern between price, time and utility or disutility; some estimates of these models are not statistically significant.

Some limitations have been found in the attempt of measuring welfare, in particular it may be presumed that the models are not performed well within a complex specification. The assumption of mid-points might also influence the results of complex specification; especially when the power term is introduced into the some coefficients. It may be also mentioned that given the nature of the dataset a nested model may be carried out since the taxi alternative may be correlated with the choice of private cars. Finally, the results presented for the Peruvian case of study might

cast light on future investigations for the detection of income effects and welfare measurement in the Peruvian transport projects appraisal.

The next table summarises the findings following the manner of Jara-Diaz and Videla, it is found that the marginal utility of income decreases as income increases.

Estimates	JD&V	Income at three levels			Income at two levels	
		300-800	1,250-2,500	3,500-10,000	300-800	1,250-10,000
ASC_{car}	3.65	2.84	1.86	2.06	2.84	1.81
($t - test$)	13.01	6.17	7.55	3.97	6.17	9.44
ASC_{taxi}	-1.89	-2.2	-2.7	-2.2	-2.2	-2.46
($t - test$)	-14.31	-11.39	-12.51	-4.52	-11.39	-14.11
β_{time}	-0.0237	-0.0312	-0.0259	-0.064	-0.0312	-0.0276
($t - test$)	-6.07	-6.44	-4.14	-3.7	-6.44	-5.01
β_{cost}	-0.238	-0.279	-0.0794	-0.0876	-0.279	-0.0772
($t - test$)	-7.58	-6.00	-2.13	-1.16	-6.00	-2.62
β_{cost}^2	0.00728	0.0096	0.00231	0.00230	0.0096	0.00223
($t - test$)	6.00	4.86	1.81	0.71	4.86	2.15
$\bar{y}_{average\ weighted}$	1,302.07	337.69	632.55	331.83	337.69	964.38
$\bar{C}1 (average)$	11.754	11.75	11.75	11.75	11.75	11.75
$\bar{C}2 (average)$	7.537	7.54	7.54	7.54	7.54	7.54
$\bar{C}3 (average)$	1.067	1.07	1.07	1.07	1.07	1.07
Diminishing effect ($\beta_{cost}^2 \cdot \bar{y}$)	9.48	3.24	1.46	0.76	3.24	2.15
λ_{c1}	0.15	0.17	0.05	0.06	0.17	0.05
λ_{c2}	0.18	0.21	0.06	0.07	0.21	0.06
λ_{c3}	0.23	0.27	0.08	0.09	0.27	0.07
$\bar{\lambda}_{weighted\ by\ market\ share}$	0.22	0.26	0.07	0.08	0.26	0.07
$\bar{\lambda} \cdot \bar{y}$	287.42	86.53	46.76	27.26	86.53	69.35

The table below shows that the marginal utility is statistical different among groups:

Income groups	Difference in $\bar{\lambda}_j$	\bar{t}
$(Y_{300-800})(Y_{1,250-2,500})$	0.182	3.300
$(Y_{300-800})(Y_{3,500-10,000})$	0.174	2.120
$(Y_{300-800})(Y_{1,250-10,000})$	0.184	3.618

1. INTRODUCTION

In the last forty years academic research in discrete choice models with foundations in microeconomic principles has developed an attractive framework for the detection of income effects. Nevertheless, a common practice is to assume that the specification of incomes within models, when calculating demands or even welfare measurement, is innocuous. Large investments in transport sector nowadays entail aggregation of benefits across individuals; this latter leads to require sophisticated tools and deep analysis for accurate measures. In this context, a mis-specification of incomes in transport models may potentially distort outcomes and unintentionally conduct to non-optimal decisions.

1.1 MOTIVATION

Since the theoretical work of Small and Rosen¹ (1981), applications in discrete choice models to welfare analysis in transportation sector have taken place in the academic ground. The underlying assumption in S&R is the absent of income effects; however, this assumption may result inappropriate when income effects are prevalent and/or the specification requirements are not observed (Batley and Ibanez, 2010).

Although most of the academic research suggest taking into account the income effect, especially when aggregating across individuals; in practice this may not occur. Hence empirical tests on the detection of income effect in transport models might encourage introducing more attention to the role of income. Within this line, the present study pursues to contribute with some empirical testing on the attempt of detecting income effects in discrete choice models.

1.2 RESEARCH OBJECTIVES

The objectives of this dissertation come from the theoretical and empirical approaches. From the theoretical field, this study aims to examine the relevant literature of income effect in discrete choice models so as to provide a straightforward and illustrative document in this subject. The empirical objective pursues to detect the

¹ Henceforth S&R.

presence of income effects in the Peruvian case of study so as to contribute with some insights of the role of income for future transport investment appraisals in this country.

1.3 RESEARCH APPROACH

The methodology developed in this dissertation is based on using revealed preferences information from the urban transport survey carried out in 2004. This information is part of a commissioned study funded by the local capital city and central government of Peru. The revealed preference survey gathers information concerning the preferred modes of transport of the citizens of Lima Metropolitan along with their alternatives modes. This manner of collecting information has enabled to apply a discrete choice framework in the present study. Thus, on the basis of this dataset some tests –suggested by the relevant literature in this ground– have been performed under the hypothesis that income effects cannot be neglected.

One first group of tests have been done to identify the presence of income effects in transport mode. Then a second group of test has been carried out by considering distinct specifications forms that theoretically hold path independence on income, prices or both along with other specifications which do not hold the independence property. One last set of tests has been done to cast light on the measure of welfare for the urban metro of Lima Metropolitan based on the framework of the logsum.

1.4 OVERVIEW OF THE CASE OF STUDY

The case of study is applied for the Lima Metropolitan Area. Since the 90' the governmental policies in the transport sector has been oriented to lax some rules concerning to entry and exit, quality of service and pricing deregulation. Although these policies initially were promoted with the intention to overcome the excess of demand, then the extreme flexibility in the rules raised congestion problems, lack of incentives for investing in capital, reduction in travel safety, among other.

The central Peruvian government has resumed and undertaken some projects in the urban transport sector; in particular, projects in urban metro. The investment of these projects entails a considerable amount of resources, thus a profound analysis of the

benefits need to be carefully done. The ignorance or the mis-specification of income effects into the appraisal may distort the welfare benefits of these projects. Thus the overall benefits may not outweigh the losses in the transport sector. For this reason, the empirical tests in the present study attempt to identify the presence of income effects and potentially implications in the welfare measurement.

1.5 DISSERTATION ORGANIZATION

The present dissertation encompasses five chapters; the second chapter broadly reviews the literature developed of the microeconomics foundations of income effects and its association within the discrete choice framework. Seminal and relevant researches in this field are examined in this section. Chapter three outlines the specifications form suggested by the literature along with some empirical researches done on the detection of income effects. In chapter four, we applied some empirical testing by using data from a revealed preference transport survey. The final chapter summarises the main findings, as well as limitations and some suggestions for appraising transport projects.

2. LITERATURE REVIEW

2.1 THEORETICAL MICROECONOMIC FOUNDATIONS

2.1.1 Individual (or consumer) decision making

The microeconomic demand theory has developed its foundations on the principles of rational individual decision making. This rationality is formally expressed by a set of axioms (principally; *completeness*, *transitivity* and *the weak axiom*) and may be modelled by two distinct approaches: preference-based approach –most related with classical microeconomic theory– and choice-based approach –most related with Random Utility Maximisation (RUM) models–. Both approaches are equivalent when choices are defined for all subsets of an affordable bundle of goods and the choice-based approach satisfies the weak axiom (Mas-Colell, 1995).

On the basis of this rationality the individual's preferences may be represented by a means of a *utility function*, though two assumptions are also needed: desirability (or monotonicity) and convexity. The former regards that larger amounts of consumption

of goods are preferred to smaller ones provided that this consumption be feasible. The latter (strong) assumption is, in essence, concerning the individual's willing to make trade-off among goods (Mas-Colell, 1995); this assumption raises two features that have important implications –in particular for the present study– in the construction of demand functions, those are: homothetic and quasilinear preferences relation (or utility functions).

A final assumption is needed to represent a formal (mathematical) individual's preference relation within a utility function; this is that preferences relation be continuous (cardinal property). This assumption implies that any set of bundles include a closed upper and lower contour set (boundaries). This also guarantees that sudden preference reversal does not occur (Jehle and Reny, 2011). This assumption also enables the application of maximising and minimising mathematical formulation in the utility function. Thus on the basis of this framework the continuous utility function may be expressed by²:

$$u = u(\mathbf{x}) = u(x_1, x_2, x_n) \quad (2-1)$$

u represents a quasilinear direct utility function; $\mathbf{x}(x_1, x_2, x_n)$ is the bundle of commodities that the individual can consume³; x_n represents the numeraire commodity (money or a fictitious commodity outside the bundle chosen) which price (or coefficient) is normalised to one. To complete the analysis of the decision making framework is needed to state some properties of the set of commodities; those are: physically feasible to consume, mutually exclusive and affordable. Likewise the price-taking assumption is made. This leads to formulate the following constraint:

$$x_n + x_1 p_1 + x_2 p_2 = y \quad (2-2)$$

$$x_j \geq 0 \quad j = 1, 2, n \quad (2-3)$$

$$\forall \mathbf{x} = (x_1, x_2, x_n); \mathbf{p} = (p_1, p_2, 1) \quad (2-4)$$

² By following many studies in this ground (e.g. Small and Rose, 1981), the present document assumed a quasilinear utility function for all formulations. This utility function is drawn for the case of three commodities. The function is assumed to be twice differentiable.

³ The interpretation for the set of commodities will vary along the document whether the demand function is continuous or discrete.

2.1.2 Continuous demand function

By using the tools of the optimization theory, two notions of demand theory can be conceived from the utility function framework (above addressed). The first one arises from the Marshall's postulations that in essence can be derived from the principle that the consumer maximises her utility level by choosing an optimal set of observable goods given prices and income. Formally this approach can be illustrated by a **primal problem** as following:

$$\text{Max } u = u(\mathbf{x}) = u(x_1, x_2, x_n) \quad (2-5)$$

$$\text{s.t. } x_n + x_1 p_1 + x_2 p_2 = x_n + \mathbf{p}\mathbf{x} = y$$

$$x_j \geq 0 \quad j = 1, 2, n$$

Optimisation of (2-5) yields the uncompensated (also called Marshallian or ordinary) demand function. This name comes from the fact this demand combines implicitly price and income effects, thus there is no income compensation to keep the consumer at the same level of utility. Formally this demand function is denoted as:

$$x_j = x_j(\mathbf{p}, y) \quad (2-6)$$

Then by substituting (2-6) in (2-1), the indirect uncompensated utility function can be obtained as:

$$v(\mathbf{p}, y) = u(x_1(p_1), x_2(p_2), x_n(p_1, p_2, y)) \quad (2-7)$$

The indirect uncompensated utility function represents the maximum level of utility attainable given the prices and income. This function satisfies the Roy's identity which provides that resulting function possess the same properties⁴ required by utility maximisation (Jehle and Reny, 2011), that is:

$$-\frac{\partial v(\mathbf{p}, y)/\partial p_j}{\partial v(\mathbf{p}, y)/\partial y} = x_j(\mathbf{p}, y) \quad (2-8)$$

A second notion of demand theory comes from the postulates of Hicks that in essence demonstrates that the assumption of constant marginal utility of income is neither necessary nor sufficient to hold the properties of demand law. This demand

⁴ Continuous, homogeneous degree zero, strictly increasing in y, decreasing in p and quasiconvex in p and y.

can be formally derived from the **dual problem**; i.e. minimising the expenditure given prices and a level of utility. Formally:

$$\text{Min } x_n + x_1 p_1 + x_2 p_2 = x_n + \mathbf{p}\mathbf{x} = y \quad (2-9)$$

$$\text{s.t. } u = u(\mathbf{x}) = u(x_1, x_2, x_n)$$

$$x_j \geq 0 \quad j = 1, 2, n$$

Optimisation of (2-9) yields the compensated (also called Hicksian) demand function. This demand can be interpreted, in terms of Hicks, as the minimum of level of income to attain a specific level of utility given a set of prices. Formally this demand is denoted:

$$x_j^c = x_j^c(\mathbf{p}, \bar{u}) \quad (2-10)$$

Then by substituting (2-10) in (2-9), the expenditure function will be equal to the cost of the set of bundles \mathbf{x}^c , and formally defined as:

$$e(\mathbf{p}, \bar{u}) = x_n + \mathbf{p}\mathbf{x}^c = y \quad (2-11)$$

The expenditure function satisfies the Shephard's lemma that states that a price increase of a specific commodity increases the expenditure to uphold the level of utility, this is:

$$\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_j} = x_j^c(\mathbf{p}, \bar{u}) \quad (2-12)$$

Even though the concepts of expenditure function and indirect demand function are different; these have a close relationship (duality). This can mathematically be expressed by⁵:

$$e(\mathbf{p}, v(\mathbf{p}, y)) \leq y \quad (2-13)$$

$$v(\mathbf{p}, e(\mathbf{p}, u)) \geq u \quad (2-14)$$

Consequently, there also exists a close relationship (duality) between the uncompensated (Marshallian) and compensated (Hicksian) demands⁶, as following:

⁵ Under some conditions there exists equality between the functions. See Mas-Collel (1995) or Jehle and Reny (2011).

⁶ In equilibrium.

$$x_j = x_j^c(\mathbf{p}, v(\mathbf{p}, y)) \quad (2-15)$$

$$x_j^c = x_j(\mathbf{p}, e(\mathbf{p}, \bar{u})) \quad (2-16)$$

Despite this duality, both demands (Marshallian and Hicksian) differ in capturing the income effect of an own-price change. Whereas the Marshallian demand capture the total effect of an own-price change, the Hicksian demand captures the pure Hicksian substitution effect (Jehle and Reny, 2011). These effects can be decomposed by the Slutsky's equation by differentiating both sides of **Error! Reference source not found.** with respect to a specific price j , this is⁷:

$$\frac{\partial x_j^c(\mathbf{p}, \bar{u})}{\partial p_j} = \frac{\partial x_j(\mathbf{p}, e(\mathbf{p}, \bar{u}))}{\partial p_j} + \frac{\partial x_j(\mathbf{p}, e(\mathbf{p}, \bar{u}))}{\partial y} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_j} \quad (2-17)$$

By rearranging some terms and (2-13) considering (2-12), the Slutsky's equation can provide the substitution and the income effects:

$$\frac{\partial x_j(\mathbf{p}, \bar{u})}{\partial p_j} = \frac{\partial x_j^c(\mathbf{p}, \bar{u})}{\partial p_j} - \frac{\partial x_j(\mathbf{p}, y)}{\partial y} x_j(\mathbf{p}, y) \quad (2-18)$$

Thus the left side of the equation represents the downward sloping of the Marshallian demand (<0) or total effect, the first term of the right side is the slope of the Hicksian demand (<0) or the substitution effect, while the second term represents the income effect (>=0 if it is a normal good; <0 if it is an inferior good).

2.1.3 Discrete-continuous demand function

A first attempt in constructing a theory of discrete-continuous demand with foundations in microeconomic principles was realised by Small and Rosen⁸ (1981). Due to the increasing popularity of discrete choice in Random Utility Maximisation (RUM) models –further address in next sections– and the theoretical microeconomic gap, S&R attempted to reconcile the neoclassical consumer theory with RUM whereby demonstrating, under some conditions, that applications of welfare economics in discrete choice might be empirically computed. For that purpose, the work of S&R starts defining⁹ concepts of discreteness in demand –which is adopted in this section– and a conditional compensated (Marshallian) demand under some assumptions.

⁷ This equation represents the special case of own-price change.

⁸ Henceforth, S&R.

⁹ The S&R analysis related to welfare economics is addressed in the following sections.

The conceptual definition of discrete-continuous demand formulated by S&R –in which basis has been developed important literature thereafter; e.g. Hu (), Batley and Ibanez (2010)– states that commodities can be consumed in continuous quantities but choosing in one mutually exclusive variety or consumed in only one or two units¹⁰, i.e. having chosen the commodity, the individual consumes a positive continuous quantity of the selected commodity. This definition leads to the following formal constraint:

$$x_1 x_2 = 0; \quad (2-19)$$

and formally expressed as:

$$\dot{u}(x) = \tilde{v}_k(p_k, y) = \text{Max} \{ \tilde{v}_1(p_1, y), \tilde{v}_2(p_2, y) \} \quad (2-20)$$

Where \dot{u} is the maximum direct utility unconditional and conditional; \tilde{v} represents the indirect utility, y is the income required to achieve \dot{u} and k is the index of the commodity chosen which for this case may be 1 or 2. Thus by following S&R and considering the latter expressions, a **primal problem** for a **conditional uncompensated** (Marshallian) demand can be derived from (2-5) by an optimisation and employing Roy's identity, that yields:

$$-\frac{\partial v / \partial p_1}{\partial v / \partial y} = \begin{cases} -\frac{\partial \tilde{v}_1 / \partial p_1}{\partial \tilde{v}_1 / \partial y} = \tilde{x}_1 \text{ if } k = 1 \\ -\frac{\partial \tilde{v}_2 / \partial p_1}{\partial \tilde{v}_2 / \partial y} = 0 \text{ if } k = 2 \end{cases} \quad (2-21)$$

$$-\frac{\partial v / \partial p_2}{\partial v / \partial y} = \begin{cases} -\frac{\partial \tilde{v}_1 / \partial p_2}{\partial \tilde{v}_1 / \partial y} = 0 \text{ if } k = 1 \\ -\frac{\partial \tilde{v}_2 / \partial p_2}{\partial \tilde{v}_2 / \partial y} = \tilde{x}_2 \text{ if } k = 2 \end{cases}$$

Likewise, following Batley and Ibanez (2010) the expenditure conditional function can be expressed as:

$$e(p, \dot{u}) = \bar{e}(p_k, \dot{u}) = \text{Min} \{ \bar{e}_1(p_1, \dot{u}), \bar{e}_2(p_2, \dot{u}) \} \quad (2-22)$$

Thus the **dual problem** for the **conditional compensated** (Hicksian) demand can be obtained by optimisation of (2-9) and employing the Shepard's lemma, this is:

¹⁰ The third definition of S&R of discreteness in demand is a variant that can be modelled as the first definition.

$$\frac{\partial e}{\partial p_1} = \begin{cases} \frac{\partial \tilde{e}_1}{\partial p_1} = \tilde{x}_1^c \text{ if } k = 1 \\ \frac{\partial \tilde{e}_2}{\partial p_1} = 0 \text{ if } k = 2 \end{cases} \quad (2-23)$$

$$\frac{\partial e}{\partial p_2} = \begin{cases} \frac{\partial \tilde{e}_1}{\partial p_2} = 0 \text{ if } k = 1 \\ \frac{\partial \tilde{e}_2}{\partial p_2} = \tilde{x}_2^c \text{ if } k = 2 \end{cases}$$

These conditional compensated demands theoretically exhibit a point of discontinuity (“undefined at some crossover price”) that may produce nondifferentiability into the expenditure function (and also into the indirect utility function), though S&R demonstrate that expenditure function is differentiable (hence continuous) by proving that since (2-19) requires that either commodity 1 or 2 be zero, the utility function may be expressed as $u = (x_n, x_1, 0)$ which provides identical results that in the case of continuous demand functions and therefore ensures that expenditure function, compensated demand and indirect utility be differentiable and continuous.

2.1.4 Consumer surplus and compensating variation in continuous and discrete-continuous demand

Consumer surplus is commonly (and straightforwardly) defined as the difference between the (maximal) willingness to pay and the price market (Johansson, 1991). In geometrical terms it is calculated as the area to the left of the uncompensated (Marshallian) demand curve above the market price and given income. Formally:

$$\Delta MCS = (v(\mathbf{p}^1, y) - v(\mathbf{p}^0, y)) = - \int_{p^0}^{p^1} \sum_j - \frac{\partial v / p_j}{\partial v / y} dp_j + x_n \quad (2-24)$$

Thus (2-24) describes a variation in the consumer surplus due to a change in price j for a **continuous demand** function. This formula is derived from the basis of quasilinear direct utility function in (2-1) and applying the Roy’s identity. This change in consumer surplus can be also expressed as:

$$\Delta MCS = - \frac{1}{\lambda} \int_{v(p^0)}^{v(p^1)} dv + x_n \quad (2-25)$$

On the other hand, an equivalent measure of the consumer surplus, under some conditions, can be obtained from the basis of the expenditure function or uncompensated (Hicksian) demand. This is:

$$\Delta CV = e(\mathbf{p}^1, u^0) - e(\mathbf{p}^0, u^0) = \int_{p^0}^{p^1} \sum_j \frac{\partial e(\mathbf{p}, u^0)}{\partial p_j} dp_j \quad (2-26)$$

The latter formula describes the compensating variation that has been calculated by employing the Shepard's lemma. The conceptual intuition of the compensating variation describes the level or amount of income at which the individual should be compensated to be as better off as before the change in price.

The especial case of quasilinear utility ensures that such the consumer surplus and compensating variation equals in equilibrium. This is due to the compensating variation in income is independent of price. Formally:

$$\Delta MCS = (v(\mathbf{p}^1, y) - v(\mathbf{p}^0, y)) = v(p_1, p_2) + x_n = (e(\mathbf{p}^1, v) - e(\mathbf{p}^0, v)) = CV \quad (2-27)$$

In the case of a **discrete-continuous** demand, the consumer surplus and compensating variation can be obtained by following the premises of S&R. Thus from S&R work and following Batley and Ibanez (2010), and having defined (2-15) and (2-16) the unconditional and conditional demands can be related with the uncompensated and compensated demand functions in the following manner:

$$x_j^i(\mathbf{p}, y) = \delta_j^i(\mathbf{p}, y^i) \tilde{x}_j^i(p_j, y^i) \quad (2-28)$$

$$x_j^{ci}(\mathbf{p}, u^{i0}) = \delta_j^{ci}(\mathbf{p}, u^{i0}) \tilde{x}_j^{ci}(p_j, u^{i0}) \quad (2-29)$$

The formulations in (2-28) and (2-29)¹¹ embody δ_j^i and δ_j^{ci} which are the uncompensated and compensated discrete choice index respectively, i.e. once the commodity $j = 1, 2$ is chosen, $\delta_j^{ci} = 1$; and 0 otherwise. Thus by following the operation in Batley and Ibanez (2010), the consumer surplus and compensating variation for a single individual (defined as i) given a change in price can be expressed by:

¹¹ For the proof of the equation, see Small and Rosen (1981).

$$v_j^i(\mathbf{p}^2, y) - v_j^i(\mathbf{p}^1, y) = - \int_{p_1^1}^{p_1^2} \lambda \cdot \delta_j^i(\mathbf{p}, y^i) \tilde{x}_j^i(\mathbf{p}, y^i) dp_1 \quad (2-30)$$

$$\tilde{e}_j^i(\mathbf{p}^2, u^{i0}) - \tilde{e}_j^i(\mathbf{p}^1, u^{i0}) = \int_{p_1^1}^{p_1^2} \delta_j^{ci}(\mathbf{p}, u^{i0}) \tilde{x}_j^{ci}(\mathbf{p}, u^{i0}) dp_1 \quad (2-31)$$

2.1.5 Income effect within the microeconomic foundations

The previous theoretical review has delineated the principles of individual decision making and maximising utility. On this basis, one first role of the individual income is to function as constraint within consumer behaviour, following the reasoning that any set of consumption must be affordable for the individual. The other implication of income arises in constructing demands functions based on the consumer behaviour. The first insight was postulated for Marshall by assuming constant marginal utility of income (money) justified on the argument that the expenditure in one commodity is a small part of the whole expenditure (Marshall, 1920 in Green J., 1976). In a different calculating manner, the marginal utility of income is derived from the utility maximisation problem and represented by the λ term (the Langrange multiplier).

Variations of consumer surplus can be measured from changes in prices or income (lump sum). The former case can have two sorts of effects: substitution –due to changes in the relative price¹²– and income –due to change in the purchasing power– effects. Formally, these two effects can be observed in the Slutsky equation. In the case of a change in the individual income, this can (or not) alter the proportion of consumption of the commodities or can reduce the consumption of one (or more) commodity of the bundle. These possible results draw the income expansion path (Varian, 1992). Both changes in prices or income can produce different measures of consumer surplus due to the order in which the relative prices vary; this is also known as the **path-dependency problem** that for this case is illustrated for aggregation across commodities (Johansson, 1991).

Thus in order to preclude the path-dependency, cross price symmetry is required; i.e. $\partial x_j / \partial p_i = \partial x_i / \partial p_j$. This symmetry can be obtained from the assumption of homothetic or quasilinear utility function. This latter is assumed along the present section,

¹² The magnitude is subject to the nature of the commodity, i.e. it depends on whether the commodity is normal, inferior or Giffen.

guarantying the absent of income effects. Formally, from (2-5) the first order derivation yields that $\partial u(x)/\partial x_n = b = \lambda$. Where b is a constant term or the coefficient of the numeraire commodity normalised to one. Thus since λ represents the marginal utility of income, this derivation is in line with the Marshall's postulation¹³. The insight behind this mathematical representation relies on the reasoning that an increase in (real) income would go into the consumption of the other commodity (within the numeraire good) since the consumption of the chosen commodity represents a small part of the whole expenditure.

In this context, once preventing the presence of income effects to measure consumer surplus for uncompensated (Marshallian) demand, it turns to address aggregation across individuals so as to be able to measure welfare.

2.2 WELFARE MEASURE IN DISCRETE-CONTINUOUS DEMAND

Thus far, the literature review has been explicitly emphasised in the context of individual-level of demands and measures of consumer surplus for conditional and unconditional cases. As seen, the assumption of quasilinear utility function permits to draw measures of (Marshallian) consumer surplus regardless income effects –in an aggregation across commodities, Hicksian demand does not exhibit this problem (Johansson, 1991)–. Nevertheless, in order to measure welfare the literature provides additional assumptions to be made for both Marshallian and Hicksian demands.

It is worth starting by defining welfare on the basis of utility function. The common definition regards the summation of variations in consumer surplus across individuals as measure of welfare. This can be calculated by maximising a social welfare function on the basis of representative consumer (behavioural approach) subject to a social budget¹⁴. The first order of conditions yields that λ is equal across individuals, i.e. the social utility of an additional amount of income is the same regardless individuals; so that the aggregation of consumer surplus provides the measure of

¹³ It is worth reminding for the case of uncompensated demands, these income implications are not applied since Hicksian demands are solely subject to substitution effect. With the assumption of a numeraire good, a partial equilibrium is reached.

¹⁴ For a formally derivation, see Mas-Collel (1992).

welfare (Mas-Collel, 1992; Johansson, 1991). Hence since λ is equal across individual, the **path independency** (or linear income expansion path) holds.

2.2.1 Aggregation in discrete-continuous demand and expected demand

Having assumed a quasilinear utility function in (2-5) for the individual-level case, and since quasilinear utility function derives quasilinear demand functions (Johansson, 1991), the aggregation of demands across individuals is plausible (Mas-Collel, 1992). Thus from the notion of unconditional and conditional demands expressed in (2-28) and (2-29), the aggregation across individual for a commodity $j = 1,2$ and N population is (Batley and Ibanez, 2010):

$$X_j = \sum_{i=1}^N \sum_{j=1,2} x_j^i = \sum_{i=1}^N \sum_{j=1,2} \delta_j^i(\mathbf{p}, y^i) \tilde{x}_j^i(p_j, y^i) \quad (2-32)$$

$$X_j^c = \sum_{i=1}^N \sum_{j=1,2} x_j^{ci} = \sum_{i=1}^N \sum_{j=1,2} \delta_j^{ci}(\mathbf{p}, u^{i0}) \tilde{x}_j^{ci}(p_j, u^{i0}) \quad (2-33)$$

The formulations in (2-32) and (2-33) are deemed as aggregate demands; however, to valid the aggregation across individuals so as to formulate the expected demands, the following identities must hold:

$$\sum_{i=1}^N \sum_{j=1,2} \delta_j^i(\mathbf{p}, u^i) = \sum_{i=1}^N \sum_{j=1,2} \delta_j^i(\mathbf{p}, y^i) = N \quad (2-34)$$

$$Y = \sum_{i=1}^N y^i \quad (2-35)$$

By the introduction of representative consumer (implying that each individual exhibits the same conditional uncompensated demand) and making explicit the following identity:

$$\pi = \frac{\sum_{i=1}^N \delta_1^i}{N} \quad (2-36)$$

where $0 \leq \pi \leq 1$ ¹⁵, $Y = Ny$ and $y^i = y, \forall i$; the following **expected demand** functions can be derived from (2-32) and (2-33) (Batley and Ibanez 2010):

¹⁵ This condition implies that $x_1 x_2 \neq 0$, thus the commodities can be consumed in combination.

$$X_j = \sum_{i=1}^N \sum_{j=1,2} \delta_j^i(\mathbf{p}, y^i) \tilde{x}_j^i(p_j, y^i) = N\pi_j(\mathbf{p}, y) \tilde{x}_j(\mathbf{p}, y) \quad (2-37)$$

$$X_j^c = \sum_{i=1}^N \sum_{j=1,2} \delta_j^{ci}(\mathbf{p}, u^{i0}) \tilde{x}_j^{ci}(p_j, u^{i0}) = N\pi_j^c(\mathbf{p}, \{u^{i0}\}) \tilde{x}_j^c(\mathbf{p}, \{u^{i0}\}) \quad (2-38)$$

From (2-37) and (2-38)¹⁶, the following identity can be derived for cases of a single unit of consumption, i.e. $\tilde{x}_1 = \tilde{x}_2 = 1$, which implies that the conditional marginal utilities of price and income are equal and common for both commodities (Jara-Diaz and Farah, 1988), this is:

$$\pi_j(\mathbf{p}, y) = \frac{\sum_{i=1}^N \sum_{j=1,2} \delta_j^i(\mathbf{p}, y^i) \tilde{x}_j^i(p_j, y^i)}{N\tilde{x}_j(\mathbf{p}, y)} \quad (2-39)$$

$$\pi_j^c(\mathbf{p}, \{u^{i0}\}) = \frac{\sum_{i=1}^N \sum_{j=1,2} \delta_j^{ci}(\mathbf{p}, u^{i0}) \tilde{x}_j^{ci}(p_j, u^{i0})}{N\tilde{x}_j^c(\mathbf{p}, \{u^{i0}\})} \quad (2-40)$$

It is worth to mentioning that the special case of $\tilde{x}_1 = \tilde{x}_2 = 1$ leads to interpret (2-37) and (2-38) as **probabilistic demands** that is addressed in the following subsection.

2.2.2 Probabilistic demand

Probabilistic demand can be formally derived from two conceptual grounds. The first one arises from basis of discrete-continuous demand where the special case of single unit consumed conducts towards probabilistic demand. The other ground comes from the basis of expected utility –with the use of utility maximisation and cost minimisation– formulated by Hau (1985, 1987). The distinction between these two is basically that this latter incorporate the notion of random error term –motivated for the adoption of RUM– whereas expected demand initiates the derivation from deterministic perspective.

Since the probabilistic demands in (2-37) and (2-38) are derived from the deterministic maximisation process, the following formulations relies on the basis of random error term (Hau, 1985 and 1987; Batley and Ibanez, 2010) which is aligned with the econometric specification of RUM. Thus by following the works of Hau, though in particular the derivation of B&I and the apparatus of RUM, the expected utility maximisation can be expressed:

¹⁶ Batley and Ibanez (2010) also demonstrate application of Roy's identity and Shepard's lemma.

$$E(\max\{\tilde{v}_j\}) = E(\max\{(W_1 + \varepsilon_1^i), (W_2 + \varepsilon_2^i)\}) = \log(\exp(W_1) + \exp(W_2)) \quad (2-41)$$

where \tilde{v}_j is conditional indirect utility of commodity j as in (2-20), W_j represents the deterministic utility and ε is the random term that is distributed IID Gumbel. Thus by deriving the expected utility with respect to the deterministic term, we obtain:

$$\frac{\partial E(\tilde{v}_j)}{\partial W_1} = \frac{\partial(\log(\exp(W_1) + \exp(W_2)))}{\partial W_1} = \frac{\exp(W_1)}{\exp(W_1) + \exp(W_2)} = \pi_1 \quad (2-42)$$

Then by replacing 2-42 into the partial differentiation with respect price and income so as to have the Roy's identity, the **probabilistic demand** is:

$$\frac{\partial E(\tilde{v}_j)/\partial p_1}{\partial E(\tilde{v}_j)/\partial y} = \frac{-\pi_1 \frac{\partial W_1}{\partial p_1}}{\left(\pi_1 \frac{\partial W_1}{\partial y} + \pi_2 \frac{\partial W_2}{\partial y}\right)} = \pi_1 \quad (2-43)$$

In order to valid this latter result, some assumptions that are postulated in Batley and Ibanez (2010) need to be complied. This will be addressed in the next subsections.

2.2.3 Welfare measure on the basis of discrete-continuous demand

One first insight of aggregate consumer surplus or welfare measure for a given change in price is formulated by S&R. Although the authors begin the derivation from the notions of discrete-continuous demand and compensated demand, they end up employing the notions of probabilistic demand and uncompensated (Marshallian) demand. Despite of it, a formal expression can be obtained following the manner in (2-30), (2-31) and (2-36):

$$\Delta E(v) \approx - \int_{p_1^1}^{p_1^2} \left[\sum_{i=1}^N \sum_{j=1,2} \lambda \cdot \pi_j^i(\mathbf{p}, y) \tilde{x}_j^i(\mathbf{p}, y) \right] dp_1 \quad (2-44)$$

where λ is the marginal utility of income.

2.2.4 Welfare measure on the basis of probabilistic demand

S&R in their attempt to derive a welfare measure, they apply the Roy's identity to the conditional demand, and following the notation in B&I, we obtain:

$$\Delta E(v) \approx - \int_{p_1^1}^{p_1^2} \frac{N}{\lambda(y)} \pi_1(\mathbf{p}, y) \frac{-\partial W_1 / \partial p_1}{\partial W_1 / \partial y} dp_1 \quad (2-45)$$

Then by considering the assumption of S&R, (2-45) can be re-expressed as:

$$\Delta E(v) \approx - \frac{N}{\lambda(y)} \int_{W_1^1}^{W_1^2} \pi_1(W_1, W_2) dW_1 \quad (2-46)$$

The formula in (2-46) corresponds to the S&R's welfare measure, derived from the probabilistic demand for a change in the indirect deterministic utility. This S&R formulation constitutes an approximation of the Hicksian consumer surplus from the basis of the Marshallian consumer surplus; hence it explicitly assumes that λ is approximately independent of the price and income effects are negligible due to the irrelevance of the discrete commodities for the individual. These assumptions are equivalent to two of the five formal postulations in Batley and Ibanez (2010).

2.2.5 Income effect in probabilistic demand

Thus far some formulations have been constructed from the works of S&R and B&I without specifying the necessary assumptions to hold the postulations. Thus in the context of a probabilistic demand, B&I identify five assumptions to hold conditions so as to convert expected demand to probabilistic demand and the properties of this latter as well as to provide tractability in measuring welfare, as following:

Assumption I: unit conditional demand for commodity 1 and 2; i.e. $\tilde{x}_1 = \tilde{x}_2 = 1$.

Assumption II: for each commodity (or alternative), equivalence in absolute terms between the conditional marginal utility of income and price; i.e. $-\partial W_1 / \partial p_1 = \partial W_1 / \partial y$, $-\partial W_2 / \partial p_1 = \partial W_2 / \partial y$

Assumption III: common conditional marginal utility of income across commodities; i.e. $-\partial W_1 / \partial y = \partial W_2 / \partial y = \lambda$.

Assumption IV: common conditional marginal utility of price across alternatives; i.e. $\partial W_1 / \partial p_1 = \partial W_2 / \partial p_2$.

Assumption V: the conditional marginal of income, λ , is independent of prices of commodities 1 and 2; i.e. $\partial \lambda / \partial p_1 = \partial \lambda / \partial p_2 = 0$.

Where assumptions I and II interrelates, assumptions II and III imply IV and assumptions II and IV imply III. Then it can be said that three independent assumptions (I, II and V). In this regard, assumption I –as mentioned previously– and therefore assumption II allows the transition from expected demand to probabilistic demand, so that formulations made in expected demand may be also applicable to probabilistic demand.

It is also theoretically recognised that individual demand functions are subject to comply with symmetric, homogeneity, negativity and the namely ‘adding up’ properties. Thus since the probabilistic demand adopted the representative consumer notion, these properties hold under the assumptions identified in B&I. Some of these properties have implications in some extent in treatment of income effect. Negativity property, for instance, entails that a (positive) income effect ought to be outweighed by a (negative) substitution effect. Moreover, homogeneity property implies that marginal utility of income should be homogeneous of degree minus one in prices guarantying path independency across commodities. Finally, with the assumption V income effect in RUM could be precluded.

2.3 DISCRETE CHOICE AND RANDOM UTILITY MODELS (RUM)

2.3.1 Discrete choice models

Discrete choice conceptually is defined as situations where a decision maker (or agent, e.g. individual, company) faces a (or set of) choice(s) over time, among a set of options¹⁷ (Train, 2009). An individual, for instance, chooses their mode of transport to commute or shopping from a set of options: car, taxi, bus, so forth. Airlines (companies) also choose their routes for a given origin and destination. The operational features of a discrete choice situation have been plausible to be generated through the tools of the statistical theory which has facilitated the modelling of discrete choices (i.e. discrete choice modelling).

The conjunction of discrete choice situation with statistical models applied into the individual behaviour initiated in the mathematical psychology field. The foundations of this theory were developed during the 1920’s; one of the pioneer authors was

¹⁷ The analysis of discrete choice entails two tasks: specification of the behavioral model and estimation of the parameters of the model (Train, 2009).

Thurstone (1927) who introduced a ‘Law of Comparative Judgment’ which incorporates a normally-distributed random error term in modelling physical strength of stimulus with the psychological perception of the individual; this is formally expressed for a pair of alternatives as $\Pr(1) = \Phi(W_1 - W_2)$ which is known as choice probability and later termed as ‘probit model’ (McFadden, 2010).

Luce (1959), later on, extended the binomial model to a broader range of choices so as to create the framework of the Probabilistic Choice System (PCS). Luce’s first contribution was to develop the axiom of Independence from Irrelevant Alternative (IIA) that asserts that relative probability of choosing one commodity over other (k and j) is the same (or constant) regardless the availability of other commodities (alternatives); this latter yields the **multinomial logit model**, formally expressed as:

$$\Pr(k) = \frac{w_k}{\sum_i w_i} = \ln \left(\frac{\Pr(k)}{1 - \Pr(k)} \right) = \frac{e^{w_k / \sum_i e^{w_i}}}{e^{w_j / \sum_i e^{w_i}}} = e^{w_k - w_j} \quad (2-47)$$

2.3.2 Theoretical foundations of RUM

Marschak (1960) and Block and Marschak (1960) introduced the conceptual framework of discrete choice models into the economics field, specifically in the neoclassical theory of individual decision-making. Thus the authors conceived a framework conceptually defined as a probabilistic representation of the individual decision-making derived from repeated choices of individual’s preferences for a given finite choice set (Batley, 2013); this theoretical conception of probabilistic choice remained consistency with the utility maximisation theory and therefore gave rise to the framework of Random Utility Models.

The theoretical notion of RUM aligned with the utility maximisation theory proposes that the individual select a choice that maximise their utility at the time of the choice situation under a probabilistic mechanism (Luce and Suppes, 1965). Thus on the basis of theory economic the utility can be replaced by the indirect utility to which is specified by a deterministic component and an error random term –that introduces the probabilistic feature–. This interpretation is formally defined as:

$$\Pr[U_{nk} > U_{nj}, \forall i \neq j] = \Pr[w_{nk} + \varepsilon_{ni} > w_{nj} + \varepsilon_{nj}, \forall k \neq j] \quad (2-48)$$

Luce and Suppes (1965) showed the formal demonstration derived by Holman and Marley in which is proven that the random components of any strict utility model (i.e. choices probabilities different than 0 and 1) are independent (i.e. ε_{nk} is distributed type I extreme value), whereby the logit model could be derived with proper choice probabilities. Thereafter, McFadden (1974) demonstrated that the “(conditional) logit formula (multinomial logit) for the choice probabilities necessarily implies that unobserved utility is distributed extreme values” (Train, 2009). On these bases the probability density function and cumulative function are:

$$f(\varepsilon_{nj}) = e^{-\varepsilon_{nj}} e^{-e^{-\varepsilon_{nj}}} \quad (2-49)$$

$$F(\varepsilon_{nj}) = e^{-e^{-\varepsilon_{nj}}} \quad (2-50)$$

One major contribution of McFadden’s work was to introduce randomness from cross-sectional variation across population rather than a single decision maker. His other major contribution was to introduce observable attributes (components of the alternatives) into discrete alternatives¹⁸ (Lewis, 2012). These formulations were performed empirically in a research of modal share for the San Francisco’s Bay Area Rapid Transit (BART) that produced a better prediction than the conventional gravity models¹⁹.

As mentioned, McFadden’s work turned probabilistic postulations into econometric specifications so as to carry out empirical work on the basis of RUM. McFadden postulates in line with the theory developed in RUM that the utility function can be expressed as nonstochastic component that reflects representative tastes of the population –then this component is interpreted as a systematic or expected utility (Batley, 2017)– entails a vector of measured attributes and error random term that referred as a stochastic and entails the idiosyncrasies of the representative in tastes –or unobserved factors–. Thus by following (2-48):

$$\Pr[\varepsilon_{nk}(s, x) - \varepsilon_{nj}(s, x) < w_{nk}(s, x) - w_{nj}(s, x)]; \quad (2-51)$$

¹⁸ For this work, McFadden won the novel prize in 2000.

¹⁹ The McFadden research estimated a 6.3 percent of mode share while the actual share was 6.2 percent.

where, in terms of McFadden, s represents the vector measured attribute and x is the vector of attributes. Then in order to determine the choice probability, (2-51) is integrated and considering (2-49), we have:

$$\Pr(nk) = \int \delta \left(\varepsilon_{nk}(s, x) < \varepsilon_{nj}(s, x) + w_{nk}(s, x) - w_{nj}(s, x) \right) f(\varepsilon) d\varepsilon_{nk}; \quad (2-52)$$

where δ is an indicator function of values 1 or 0. Then (2-52) can be converted into a logit choice probability that is equivalent to (2-47) implying the error term is extreme value distributed (McFadden, 1974); this is:

$$\Pr(nk) = \frac{e^{w_{nk}}}{\sum_j e^{w_{nj}}} \quad (2-53)$$

From the basis of McFadden work several more advanced models have been able to be developed (Cramer, 2013). These models have been categorised as generalised extreme valued models, mixed, mixed probit among other (Train, 2009)²⁰.

2.3.3 Consumer surplus and welfare with RUM

Under the conceptual framework of RUM, in particular under the assumptions of logit models, the consumer surplus can be computed. Starting from the definition that the consumer surplus is the monetary utility the individual receives in the choice situation (Train, 2009), thus by following the notation in (2-45) this can be expressed:

$$E(CS_n) \approx \frac{1}{\alpha_n} E[\max_j (W_{nj} + \varepsilon_{nj})]; \quad (2-54)$$

where α_n represents that marginal utility of income corresponding to the individual n , which is constant with respect to income. Thus by interpreting the S&R in (2-46) with (2-54) where ε_{nj} is IID extreme value, the consumer surplus in multinomial logit model yields the logsum (Williams, 1977; Small and Rosen, 1981); this is:

$$E(CS_n) = \frac{1}{\lambda_n(y)} \ln \left(\sum_{j=1,2} e^{W_{nj}} \right) + C \quad (2-55)$$

²⁰ Further discrete choice models have been developed over years on the basis of multinomial logit, though these are not part of the scope of the present research. See Cramer (2003), Train (2009).

In (2-55) the constant term C represents the unknown measure that equals the true consumer surplus. Thus by following Train (2009), the variation of consumer surplus can be computed as:

$$E\Delta(CS_n) = \frac{1}{\lambda_n(y)} \left[\ln\left(\sum_{j=1,2} e^{W_{nj}^1} \right) - \ln\left(\sum_{j=1,2} e^{W_{nj}^0} \right) \right] \quad (2-56)$$

The critical assumption in (2-56) is that $\lambda_n(y) = c$; where c is a constant term (or a scalar) i.e. independent of income²¹. This latter assumption implies the implementation of a representative individual so as to assure no income effects (however, the form how the income expansion path is manifested is arguable (Daly, A. et.al (2008)). It is to note that whether the change in consumer surplus is large, the proposed approximation may be severely biased (McFadden, 1999); to correct this problem McFadden proposes an algorithm (Monte Carlo Markow Chaing). Nevertheless, Karlstrom (1998, 2001) suggests a method to welfare measure in presence of income effect that be addressed in the next section.

2.3.4 Income effects in RUM

In their econometric specification Small and Rosen (1981) introduce an additive function of income within the indirect utility. McFadden (1981) also introduces the concept of the Additive-Income RUM (called AIRUM) demonstrating from the theory of probabilistic choice that income is negligible in the choice probabilities since it impacts in an equal manner to the conditional utilities of the commodities or alternatives. This can be expressed by following the notation in S&R and the indirect utility define in (2-20):

$$\tilde{v}_k^i(p_k, q_k, y^i) = z^i(y^i) + W_k(p_k, q_k, y^i; S^i) + \varepsilon_k^i \quad (2-57)$$

By applying the probabilistic framework of RUM in (2-57), we have:

$$\Pr_k = \Pr(\tilde{v}_k^i > \tilde{v}_j^i) = \Pr(z^i(y^i) + W_k + \varepsilon_k^i > z^i(y^i) + W_j + \varepsilon_j^i) \quad (2-58)$$

²¹ Despite of using the notation in S&R, it is to remark that $\lambda_n(y)$ in (2-56) is a constant term.

$$\Pr_k = \Pr(\varepsilon_j^i - \varepsilon_k^i < (z^i(y^i) + W_k) - (z^i(y^i) + W_j)) = \Pr(\varepsilon_j^i - \varepsilon_k^i < W_k - W_j)$$

(2-58) indicates that the inclusion of income does not alter the choice probabilities. In measuring welfare, however, income may not in some cases be assumed away; in particular in those cases of large changes in welfare effect or in assessing large infrastructure projects that may change the pattern of accessibility (Karlstrom, 2001). For this purpose, Karlstrom (1998, 2001) introduces a method of measuring welfare with a RUM framework in presence of income effects, i.e. non-linear income effects. Although this formulation cannot hold some microeconomic conditions as to Slutsky symmetry and integrability²² (Daly, A. et.al 2008), its method is practical and tractable for measuring welfare.

Thus the Karlstrom's method conceptually defines a given expected expenditure level so that the individual is as well off before the change in price or quality. The method also sets homogenous groups of individuals that are distinguished by unobservable factors expressed in the error term with type I extreme value distribution. In the case of two commodities (namely k and j) and given an increase in price j , the homogeneous individuals can be group as²³:

- Group A: individuals who choose commodity k both before and after an increase in price.
- Group B: individuals who choose commodity j both before and after an increase in price.
- Group C: individuals who choose commodity k before an increase in price, and commodity j after.

Given these three scenarios of grouped individuals, a post evaluation may be inferred so as to keep individuals as the given level of expenditure, i.e. to keep them as well off as before the increase in price j ; this is:

- Individuals from A would not require any compensation for the price increase of j .
- Individuals from B would require a compensation in the magnitude of the increase in price j . It is to note that discrete choice would remain constant.

²² To recover a consumer's utility function from the demand function (Jehle and Reny, 2011).

²³ The succeeding exemplification follows the insights and the manner developed in Lewis (2012).

- Individuals from C would require some compensation between 0 and the magnitude of j (name this difference as d).

It is to mention that individuals who worth commodity j above d would not switch preferences, they would be set in group B; likewise those who worth below zero, they would not have chosen j in first place. These scenarios can be illustrated as:

$$E[m] = y^0 \Pr(\text{in } A | y = y^0) + (y^0 + d) \Pr(\text{in } B | y = y^0 + d) + C_c; \quad (2-59)$$

where $E[cv] = y^0 - E[m]$, C_c is not an uniform compensating variation, though compensating variation in groups A and B are. The error term of commodity j can be found within the deterministic range:

$$W_k(p_k) - W_j(p_j^0) < \varepsilon_j < W_k(p_k) - W_j(p_j^1) \quad (2-60)$$

The previous exemplification can be illustrated in the formally expression postulated by Karlstrom (2001) which provides the general case of a compensating variation for a variation in price or quality; this is:

$$E[m] = -\sum_i \int_{\underline{u}}^{u_{ii}} y dP_i(y) = \sum_i \left\{ u_{ii} P_i(u_{ii}) - \int_{\underline{u}}^{u_{ii}} y dP_i(y) \right\}; \quad (2-61)$$

where i indexes the n commodities or alternatives and $P_i(y)$ represents that choice probability of i given the vector of deterministic components of utility as:

$$P_i(y) = P_i(g_1(y), \dots, v_i^0(y^0), \dots, g_j(y)); \quad (2-62)$$

where u_{ii} is the amount of income that the individual need to be compensated when she chooses i so as to be as well off as before the variation in price or quality; $\underline{u} = \min_k u_{kk}$; $v_i^s(\cdot)$ is the deterministic term of the conditional indirect utility function of commodity j ; and $g_i(\cdot) = \max[v_j^0(y^0), v_j^1(y)]$.

The formula in (2-61) corresponds to the Theorem 1 of Karlstrom that is applicable to additive RUM models with or without GEV distribution. Some difficulty may be found in calculating the function of P_i ; however, since the error terms are unchangeable, a maximum function before and after is correctly applied (Daly, A. et.al, 2008). The first empirical application of the Karlstrom's theorem is seen in Franklin (2006) in which results of linear model fitted the data substantially better than the non-linear one. In

Zao, Kockelman and Karlstrom (2008) find out that the mean value of the compensating variation depends on the assumption as to the correlation of the error terms (Daly, A. et.al, 2008). These theoretical and empirical considerations are to be accounted for in the case study section.

3. ECONOMETRIC SPECIFICATIONS OF INCOME IN DISCRETE CHOICE MODELS

Since the work of McFadden in 1974, the apparatus of RUM has been employed in the empirical field. To accomplish the empirical testing in RUM, some econometric specifications have been proposed in the literature. In particular, the inclusion of income within the model specification has been broadly discussed (McFadden, 1981; Small and Rosen, 1981; Jara-Diaz and Videla, 1989; Batley and Ibanez, 2010); the purpose of it has been to harmonise or develop theoretical consistency between the microeconomic foundations and the apparatus of RUM. As reviewed in the literature section, although the presence of income does not alter the choice probability between alternatives –under some specifications–, this may lead to inaccurate measure of welfare when aggregating across individuals. Thus, this section attempts to address econometric specifications in discrete choice models with income effect that have been realised theoretically or empirically. Some specifications in this section will serve to be emulated in the case of study.

3.1 SPECIFICATION OF INCOME IN DISCRETE CHOICE MODELS

3.1.1 Linear specification of residual income

The residual income introduced by McFadden (1981) enables to specify, for practical purposes, a linear relationship between price and income. Thus the utility function with a residual income is given by (Furlong, K., 2011)²⁴:

$$v(y, p_j, x_j) = \alpha h(y, p_j) + \beta f(x_j); \quad (3-1)$$

²⁴ The demonstration is taken from the author work. Indeed in what following most of the demonstrations are taken from that source

where $\alpha h(y, p_j) = \alpha(y - p_j)$ is econometric specification. When assuming a logit model and applied probabilities to the utility functions, the constancy of the marginal utility of income can be proven as followed:

$$Pr_j = \Pr[\varepsilon_k - \varepsilon_j < v(y - p_j, x_j) - v(y - p_k, x_k)] \quad (3-2)$$

$$Pr_j = \Pr[\varepsilon_k - \varepsilon_j < [\alpha(y - p_j) - \alpha(y - p_k)] + [\beta f(x_j) - \beta f(x_k)]]$$

$$Pr_j = \Pr[\varepsilon_k - \varepsilon_j < [\alpha(p_k - p_j)] + \beta[f(x_j) - f(x_k)]]$$

The (3-2) proves that the marginal utility of income is constant given by α .

3.1.2 Non-linear specification of the residuals income term

The non-linearity in specifications can be overcome by specifying the log-form to the residual income, this is: $\alpha h(y, p_j) = \alpha \log(y - p_j)$. This sort of specification has been developed in some works (Berry et al., 1995; Petrin, 2002; Franklin, 2006). Thus by following the manner in (3-2), we have that:

$$Pr_j = \Pr[\varepsilon_k - \varepsilon_j < \alpha[\log(y - p_j) - \log(y - p_k)] + \beta[f(x_j) - f(x_k)]] \quad (3-3)$$

The marginal utility of income in (3-3) is given by: $\alpha \cdot 1/(y - p_j)$. Nevertheless this specification may be problematic when $y < p_j$ as seen in Berry et.al (1995) and Petrin (2002) since in those cases the automobile is priced larger than the perceived income; conversely this is not an issue in a modal choice since $y > p_j$ where prices are relative smaller in the individual's budget.

3.1.3 Interaction among income and/or cost

Another approach of introducing income effect is to assume α constant and allow it vary across different levels of income. To represent this, the function form ought to be $\alpha^* h(y, p_j) = \alpha^*(y - p_j)$; where α^* varies as (Furlong, K., 2011):

$$\alpha^* = \begin{cases} \alpha_1 & \text{if } y \leq y_1 \\ \dots & \\ \alpha_n & \text{if } y \geq y_{n-1} \end{cases} \quad (3-4)$$

To interact with cost, α need to be derived from the base-value β^y so that it interacts with the cost of the alternative. This resembles the case of different alternative-specific cost coefficients.

3.1.4 Income and taste

The relationship between income and taste has been carried out from two approaches; the first regards income as proxy variable of tastes as to how income may explain preferences rather than simple affordability terms. Some works in this line can be found in Viton (1984), Mannering and Winston (1995) or Train and Winston (2007). The function form for this approach is given by (Furlong, K., 2011):

$$\alpha h(y, I_i, p_j) = \alpha_p(y - p_j) + \alpha_{p/I} \frac{(y - p_j)}{I_i} \quad (3-5)$$

Thus by considering the translation invariant property, we have:

$$\begin{aligned} \alpha_p h(y, I_i, p_j) - \alpha_p h(y, I_i, p_k) &= \left[\alpha_p(y - p_j) + \alpha_{p/I} \frac{(y - p_j)}{I_i} \right] - \left[\alpha_p(y - p_k) + \alpha_{p/I} \frac{(y - p_k)}{I_i} \right] \\ &= [\alpha_p(y - p_j) - \alpha_p(y - p_k)] + \left[\alpha_{p/I} \frac{(y - p_j)}{I_i} - \alpha_{p/I} \frac{(y - p_k)}{I_i} \right] \\ &= [\alpha_p(p_k - p_j)] + \left[\alpha_{p/I} \frac{(p_k - p_j)}{I_i} \right] \end{aligned} \quad (3-6)$$

As a result of this specification, the marginal utility of income which varies with the taste income is given by: $\alpha_p + \alpha_{p/I} 1/I_i$; where I_i represents the taste income and y the budget (financial) constraint. Viton (1985) argues that as long as the inclusion of the taste income does not regard as measured income, the specification can be consistent with the utility maximisation framework. However, Jara Diaz (1990) carries out an empirical work to determine whether income can serve as a proxy of taste or as purchasing power (second approach). The author employs the trading off of the decision-maker framework between work and leisure (Train and McFadden, 1978); the analysis with Chilean data set yields results statistically insignificant when income is performing as a proxy of taste.

3.1.5 Practical specifications on the path independence

One recent study, Batley and Ibanez (2013), outlines the path independency conditions applicable to discrete-continuous demand and probabilistic demand; this study also shows some common practical specifications that have been utilised in the literature of discrete choice models in compliance with path independency properties.

By following the manner of the authors, the specifications forms are shown in the succeeding table:

Table 3-1

Conditional indirect utility	Properties	Specification characteristic
$\tilde{v}_j = \lambda(y - p_i) + \varepsilon_j$	Path independent on income and prices	Linear residual income
$\tilde{v}_j = \beta_j(y^\alpha - p_i) + \varepsilon_j$	Path independent on income and prices	Power term on income
$\tilde{v}_j = \lambda_j(y - p_j^\alpha) + \varepsilon_j$	Path independent on income and prices	Power term on prices
$\tilde{v}_j = \beta_j(p_i/y) + \varepsilon_j$	Path dependent on prices and income	Price as ratio of income
$\tilde{v}_j = \beta_j(y - p_i)^\alpha + \varepsilon_j$	Path dependent on prices, income and numeraire	Power term on residual income
$\tilde{v}_j = \beta_j y^\alpha p_j^\gamma + \varepsilon_j$	Path dependent on prices and income	Cobb-Douglas-type

Source: Batley and Ibanez (2013).

The econometric specifications in Table 3-1 are to be carried out in the practical testing of the case study so as to identify the presence of income effects and its statistical significance.

3.2 SPECIFICATION OF INCOME IN DISCRETE CHOICE MODELS FOR WELFARE MEASUREMENT

In what follows in this subsection, the two analytic approaches suggested by Delle Site and Salucci (2011) for measuring welfare in discrete choice logit models with income effect are to be examined. These two approaches are the representative consumer and the compensating variation. It is to remark that thus far the assumption of constancy of the marginal utility of income has explicitly explored; however, in measuring welfare the errors terms in ex-ante and post choice scenarios are assumed to be perfectly correlated (Zhao et.al, 2008). Without undermining this latter assumption, the examination in the two approaches will preserve the assumption of perfect correlation in error terms between initial and final scenario.

3.2.1 Representative consumer

The representative consumer underlies the decisions-making in a homogenous population can be reproduced accurately in a single decision-maker, i.e. the utility maximisation of the representative consumer yields the aggregate demand. In a discrete choice setting the assumption of a representative consumer may seem to

violate the discreteness framework due to the simultaneous consumption of discrete alternatives. Anderson et.al (1988), however, points out that decision-making are made in an iterative and overlapping fashion. Thus a choice probability of 10% for alternative j signifies that decision-maker chooses j every ten intervals.

3.2.1.1 Expected maximum utility

The expected maximum utility can be measured from a static movement of state. This concept comes from the notion of the utility maximising alternative in an RUM framework. This can be formally expressed as:

$$EMU = E[\max_i(\tilde{v}_i^0)] = E[\max_i(w_i^0 + \varepsilon_i)] \quad (3-7)$$

To determine the income compensation that equates the EMU in the same state given a price or quality change, and assuming the deterministic component is linear income; the following equivalence need to be complied:

$$\ln \sum_i \exp[w_i(y - p_1^0, \bar{w}_i^0)] = \ln \sum_i \exp[w_i(y - p_1^1 - cv, \bar{w}_i^0)] \quad (3-8)$$

3.2.1.2 A classical interpretation

Delle Site and Salucci (2011) postulate the approach of a classical interpretation by assigning the same indirect utility function to each individual in the population to reproduce, by means of the Roy's identity, the same aggregate demands as a probabilistic demand model. Thus for a unit price in the numeraire good, adopting the Lewis (2012) illustration, the formula of Delle Site and Salucci (2011) to derive the indirect utility function is:

$$V = V \left[\tilde{v}_i \left(\frac{I}{N} - p_i^1 - cv, w_i^1 \right), i = 1, \dots, M \right] = - \sum \int \frac{e^{w_i}}{\partial w_i / \partial p_i}; \quad (3-9)$$

where I and N represent the aggregate income and the number of population. On this basis the cv is the variation of income that satisfies:

$$V = V \left[\tilde{v}_i \left(\frac{I}{N} - p_i^1 - cv, w_i^1 \right), i = 1, \dots, M \right] = V \left[\tilde{v}_i \left(\frac{I}{N} - p_i^0, w_i^0 \right), i = 1, \dots, M \right] \quad (3-10)$$

3.2.2 Expected compensating variation

The expectation of the compensating variation ($E(cv)$) attempts to preclude that the marginal utility of the numeraire good varies with the level of income, the alternative utility of the numeraire good and/or the alternative chosen (Karlstrom and Morey, 2004). Due to that fact the $E(cv)$ is integrated over the entire change, the marginal utility of income is allowed to vary over prices and alternatives, yielding:

$$\max_k[\tilde{v}_k^0] = \max_k[\tilde{v}_k^1] \quad (3-11)$$

$$\max_k[w_k(y - p_k^0, \bar{w}_k^0) + \varepsilon_k] = \max_k[w_k(y - p_k^1 - cv, \bar{w}_k^1) + \varepsilon_k] \quad (3-12)$$

While the $\max_i(\tilde{v}_i^0)$ is an expected value, the cv is an exact amount.

3.2.2.1 Karlstrom method

The technique developed by Karlstrom (1998, 2001) and exposed in the preceding section employs the concept of expected expenditure level $E(m)$ for the measure of the compensating variation (see subsection in 2.3.4).

3.2.2.2 McFadden simulation method

McFadden (1995) proposes a method –computing willingness to pay in RUM– on the basis of multinomial logit model that by simulation provides a consistent approximation of cv for a single individual –the representative consumer– in the presence of income effects. The McFadden method can be described in the following steps (Lewis, 2012):

- Make assumption regarding the number of iterations and the distributions of the errors. The number of iterations leads to the errors of $E[CV]$ to converge within an acceptable range.
- Each iteration generates a vector of the error term that each element is included in the deterministic utility.
- Derive the cv by equating utility of the utility-maximising alternative before and after price or quality variation as in (3-12).
- Average the iterations values for cv_t .

Thus the McFadden formula is given by:

$$E[cv] = \frac{1}{T} \sum_1^T cv(\eta_t) \quad (3-13)$$

3.3 EMPIRICAL STUDIES IN TRANSPORT WITH INCOME EFFECTS

3.3.1 In the detection of income effect: Jara-Diaz and Videla (1989)

In their attempt to detect income effect in RUM, Jara-Diaz and Videla (1989) postulate a framework by deriving a formula so as to measure the size of the income effect in a logit model; the residual income is included in a linear manner, though its coefficient depends on the alternative. The authors carry out an empirical testing for data from the Corridor in Santiago, Chile; whereby capturing income effect phenomenon from an actual data. This framework starts from the following maximisation process:

$$v^* = \max v(\mathbf{p}, y - p_j, \mathbf{q}_j) = v_1(\mathbf{p}, y - p_j) + u_2(\mathbf{q}_j) \quad (3-14)$$

Thus the authors employ the following specification in the econometric model:

$$v_i = c_i + \beta_1 p_i + 1/2 \cdot \beta_2 p_i^2 + u_2(\mathbf{q}_j); \quad (3-15)$$

where c_i is the alternative specific constant; the choice probability is given by the logit form, this is: $\pi_i = e^{v_i} / \sum_i e^{v_j}$; and the marginal utility of income is: $\lambda_i = \partial v_i / \partial y = -\beta_1 - \beta_2 p_i$. Thus by inferring the sensitivity of income depends on the significance of β_2 .

4. CASE OF STUDY

4.1 THE URBAN RAILWAY (METRO) TRANSPORT SYSTEM PROJECT IN LIMA METROPOLITAN AREA

Projects in transport sector generally entail considerable public and private investments –for instance: green field ones–. Transport projects are conventionally appraised by standards rules or guidelines made by the public entities in charge which commonly attempt to measure and add all possible benefits (quantitate or qualitative) by assumptions of some method of conversion so as to approximately

identify whether benefits may offset the project costs. A second approach –although it is less used in practice, it tends to be theoretically more suitable for welfare measurement– is on the basis of utility theory. This latter approach may lack of consistency in measuring welfare when income effects are presence, though methods and specifications have been proposed for the treatment of non-linear income effects.

Lima Metropolitan Area²⁵, formed by the conurbation of Lima (the capital city of Peru) and Callao (seaport region), has been encountered serious issues, especially those in quality service and external costs²⁶, in the urban transport sector. As in many experiences in developing countries, Lima was not the exception, the heavy social cost in transport can be examined from different dimensions and particularly attributed to continuously increase in travel time; the lack of economic-and-social regulation and incentives (e.g. no barriers to entry and market-price have induced to set prices below average cost and hence not to incentivise capital investment in public suppliers (Button, 2013)) associated with the vulnerable institutional framework (e.g. fragmented functions in public entities and responsibility roles in the private firms along with incapability of enforcement, among others) have indeed reinforced the poor performance in the sector (World Bank, 2013).

It is within in this context that in the last decade the Central Government of Peru resumed the evaluation of transport projects so as to plan an integrated system of massive transport. As a result in 2004, the commissioned entity in charge proposed within their alternatives an urban metro system to enhance the facilities in urban transport infrastructure. The four railway projects proposed have been undertaken by the Peruvian government and nowadays, based on those studies, two metro lines (“Linea 1²⁷” and “Linea 2²⁸”) have been already warranted by private firms. The appraisal of these two projects, in particular the cost-benefit analysis, gather information release in 2004 whereby formulating the benefits of these projects. Nevertheless, none testing as to how incomes influence in welfare measurement has been made on the approach of discrete choice models even when the collected

²⁵ Henceforth: Lima Metropolitan.

²⁶ Although it is not part of the scope of the present document, it is to recognise that transport sector treated in a holistic perspective leads to account for social, environment, political issues among others. Without ignoring the relevant of those approaches, this document regards in its analysis the issues related to time and cost.

²⁷ Currently the whole extension of this line is in operation phase.

²⁸ This line has been bid in March 2014, and will likely be under construction in September 2014.

information in some extent exhibit properties for this sort of research. Hence one aim of this case of study is to illustrate on these bases the role of incomes in the metro system project.

4.2 BACKGROUND OF PUBLIC TRANSPORT IN LIMA

4.2.1 Brief situational description

During the first years of the 90' some policies and reforms oriented to the economic deregulation and transfer of ownership in some sectors were introduced by the Peruvian Central Government. In 1991, the urban transport sector was deregulated in terms of pricing ("free market-pricing"), quality (lax in restrictions of the vehicles' characteristics) and free entry and exit; although this reform was motivated to overcome the deficit in supply (or the excess of demand caused by the migration and the urban extension of the city), some years later the situation turned into a crisis of oversupply, poor quality in service and environmental issues (Bielich, 2009). This state continued over years²⁹, though in the first years of 2000' the Central Government opted to design a sustainable urban plan and resume mega projects – abandoned since 1989– in rail infrastructure as a means of mitigation of the problems in Lima's urban transport.

4.2.2 Description of the projects

An important study, the Master Plan of Urban Transport for Lima Metropolitan³⁰ in 2004, was commissioned by the Peruvian Central Government to evaluate and diagnose the actual situation in the urban transport system of Lima, specially emphasised in public sphere. Within the proposals of the study, an integrated railway (metro) system was introduced which consist in the construction of four routes of metro to connect the suburban areas to the centre of Lima. This study has served to delimitate the proceeding engineering evaluation in the "Linea 1" and "Linea 2" of metro of Lima, yet the collection of information through revealed and stated preferences surveys has been used for the measurement of benefits.

The appraisal of both projects was conducted by conventional methods of cost-benefit analysis. The quantitative amount of benefits was principally composed by

²⁹ Nowadays the local government of the Lima city has undertaken a reform.

³⁰ Elaborated by JICA.

travel time savings accompanied with superior services quality in travel time. Some of these characteristics are show in the following table:

Table 4-1

Characteristics	'Linea 1'	'Linea 2'
Subsidised PPP contract	Direct annually payments due to the network and service operation.	Direct payments due to construction and annually payments for the operation.
Investment aprox. (USD)	900 million	6.5 billion
Demand estimated 2018 (daily)	231,480 passengers	304,459 passengers
Demand estimated 2020 (daily)	240,424 passengers	662,346 passengers
NPV (USD)	105,710	759,000
Rate of return	12.32%	11.33%
Ratio B/C	n.a	1.15
Discount rate	12%	9%
Value of travel time per hour (USD)	n.a	2.41

Sources: Proinversion.

By examining the cost-benefit results of the feasible studies, it can be identified that non analysis has been made as to how income may influence the mode choice by, for example, the size of the income elasticity, or how the measurement of welfare may be altered in the presence of income effect; thus it can be inferred that constancy in the marginal utility of income was implicitly assumed. This strong assumption may be countenanced in a context where the analysed population may exhibit homogenous characteristics (i.e. rather homogenous level of income); however, income levels in Lima seems to differ significantly between groups of population, as seen in the proceeding subsection.

4.2.3 Incomes in Lima Metropolitan

Since the empirical work's Kuznets in 1995, many researches have been made in the attempt to study the features of income distribution in populations (Gasparini et.al, 2007). In the last decade, many studies have researched, from a directly or indirectly approach, the features concerning income distribution in Peru (most of these studies under the approach of the poverty or income inequality (Chacaltana, 2006)). An empirical research in this field for 1996-2005 period, (Portugal, 2007), has shown statistical evidence that in terms of income distribution³¹ the population in Lima Metropolitan can be characterised into two groups of individuals: low-income level

³¹ Although the study regards the personal income, this provides insights of incomes at the household level.

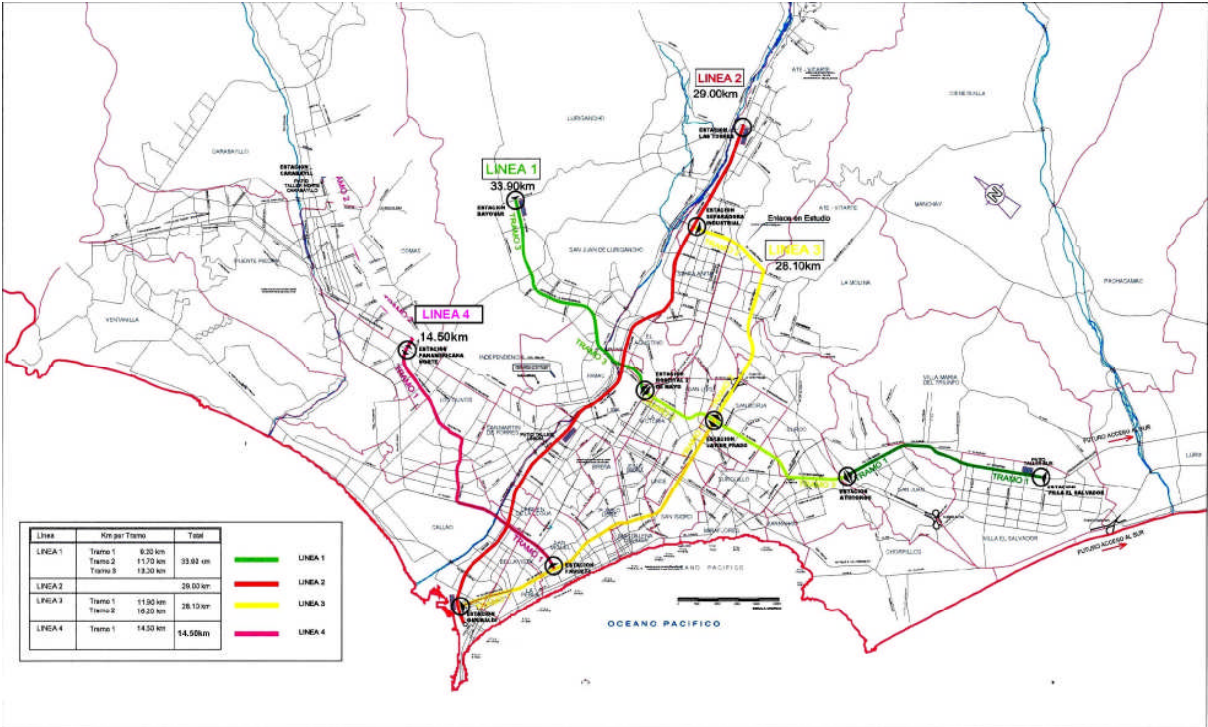
and high-income level³². One of the household surveys carried out by the commissioned study also provides categorical information regarding incomes of households along with transport expenditure by household, this is:

Table 4-2

Level of income (PEN)	Cost of transport per household				Mid-point income	Ratio cost of transport
	Min	Max	Daily	Monthly		
< = 600	0.2	204.0	5.1	128.0	300	42.7%
601 – 1,000	0.3	124.0	6.8	170.0	800	21.2%
1,001 – 1,500	0.5	168.0	9.0	225.0	1,250	18.0%
1,501 – 2,000	0.5	126.0	11.1	277.0	1,750	15.8%
2,001 – 3,000	0.5	193.0	14.2	346.0	2,500	14.2%
3,001 – 4,000	0.5	180.0	17.8	446.0	3,500	12.7%
4,001 – 7,000	1.0	131.5	26.3	658.0	5,500	12.0%
> 7,000	1.5	118.0	34.7	868.0	10,000	8.7%

Source: Master Plan of Urban Transport for Lima Metropolitan (2004).

In this regard, and with the purpose of illustrating the network metro system initially designed –see that length and route of the ‘Linea 1’ and ‘Linea 2’ which have been undertaken in the short-run have been slightly modified, when compared with their final designs–, the following figure is depicted:



Source: Master Plan of Urban Transport for Lima Metropolitan.

Figure 4-1

³² The study shows that before 1996, Lima Metropolitan can be categorised into three groups.

4.3 METHODOLOGY AND EMPIRICAL APPLICATION

4.3.1 Methodology

The methodology developed in this study is based on using the cross-sectional dataset collected in the household transport surveys (revealed preference³³) in 2004 for Lima Metropolitan so as to identify the presence of income effects by testing distinct specifications models suggested in the literature reviewed; along with this latter, an hypothetical approximation of welfare measurement for the case of metro projects is to be simulated on the basis RP data.

RP data is defined as the collection of choices that are made in actual markets or choices that actually occurred. RP data is characterised for the advantage of replicating the real market shares; however, the underlying assumption of market-equilibrium behind the RP method may not be appropriate for the assessment of non-market goods or services which required the establishment of hypothetical alternatives to capture the individuals' preferences so as to introduce innovation or potential entrants to the market. Similarly, in a RP method the revealed alternatives are likely to be non-experienced or rarely used, so attributes level may capture in some extent perceptual beliefs rather than the actual features of the alternative choices (Hensher et.al, 2005).

Due to pursuing the detection of income effect, the RP seems to be suitable for this purpose. With the RP data, it is feasible to detect income effects in the actual workings of the transport market. The discrete choice experiment can be drawn by specifying the alternatives revealed for the individual; however, it is rather likely to have unbalance in alternatives and attributes level across individuals. Even when in a set of finite and countable alternatives, some of them may be inaccessible for some individuals. On the other hand, for measuring welfare measurement, stated preference³⁴ data seems to provide suitable information for measuring any variation between an ex-ante state (real scenario, without any change) and ex-post state (hypothetical scenario, with changes in the alternatives or attributes).

³³ Henceforth: RP.

³⁴ Henceforth: SP.

The RP data collected by the commissioned study contains information regarding the actual (or most frequently) mode of transport used along with the accessible (and rarely used) mode of transport alternative. The alternatives, such private car, taxi and shared car and public transport (bus, minivan among other sort of vehicles), along with attributes such as in-vehicle travel time, cost of transport, distance between the place of origin to (bus or taxi) station and the number of interchanges among modes, were defined in the surveys; though the attributes level were revealed by the respondents. As expected the non-accessibility constraint arises for some individuals, in particular for the private car mode of transport; this effect nonetheless is capture correctly within the models.

The exercise for the hypothetical welfare measurement, on the other hand, is also based on the RP data. With the parameters estimated in the discrete choice models, an ex-post scenario in which some attributes level are to be varied is modelled through a simulation method in a spreadsheet³⁵. Since the collected information by then was based on actual conditions of 'market equilibrium', the modes alternatives such as metro or BRT (Bus Rapid Transport) were not available for users, these modes indeed constituted hypothetical alternatives for the respondents. The preferences of these non-market services were collected by SP experiment; nevertheless that dataset cannot be employed in this study³⁶.

Thus, having mentioned the essence of the dataset to be used in the present case of study, the procedure to follow can be illustrated by a diagram:

³⁵ This spreadsheet was provided in the module TRAN5281M, ITS 2014.

³⁶ The information released in the commissioned study does not provide the questions asked in the questionnaires, thus without it the conversion from raking preference to choice preference cannot be realised.

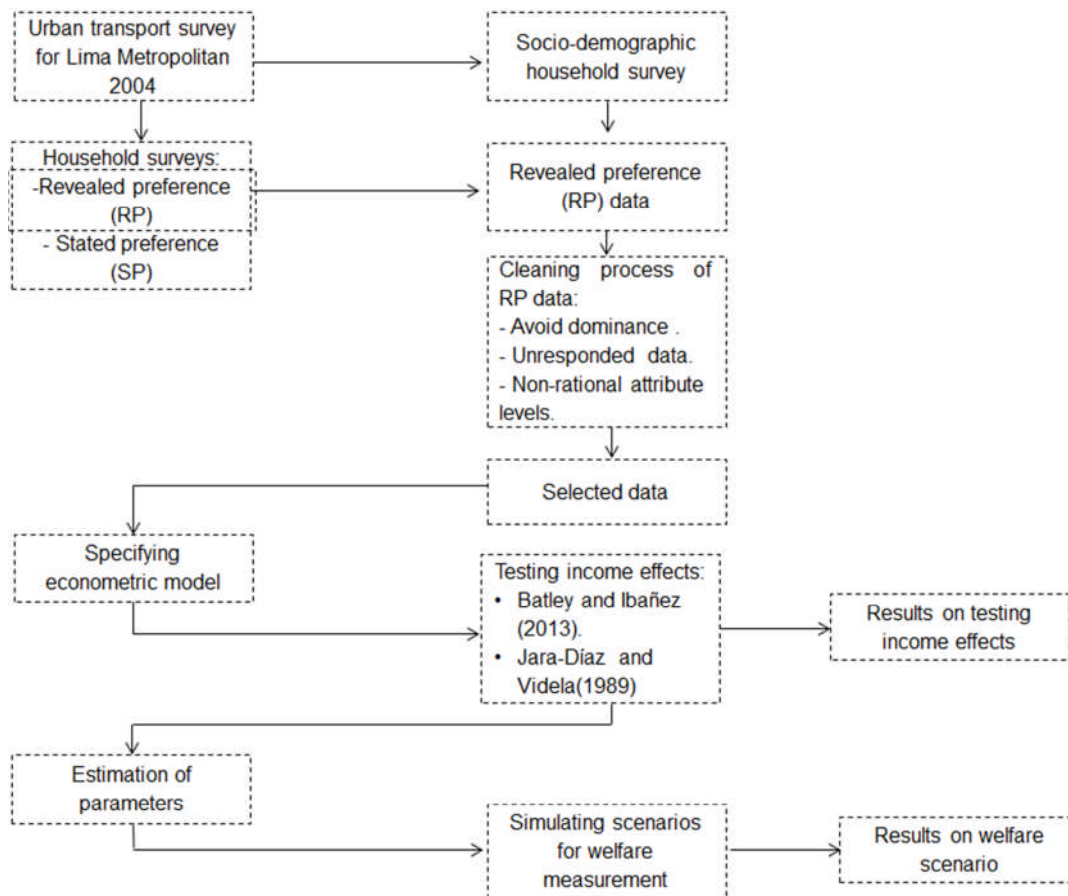


Figure 4-2

4.3.2 Surveys and dataset

The RP survey was made at household level and can be separated into two groups; the first one includes information regarding socio demographic and travel behaviour characteristics of 35,040 household samples, which contains 157,739 individuals interviewed. The second survey group belongs to those individuals who unveiled the use of any mode of transport; those were asked to reveal the attributes level of the alternative modes; this yields 8,186 respondents for distinct travel purpose. This last RP survey reflects the transport mode decision-making faced by users in a current market situation. Given the nature of the RP survey, the collected data is confined to accessible alternatives for the individuals; this latter provokes alternative-unbalance across individuals which needed a special treatment in computing the models.

Table 4-3

Attribute	Private car	Taxi	Public transport
Travel time	Minutes		
Travel cost	PEN (Peruvian currency)		

Distance to station	Metros	
Number of interchanges	Non available	Number
Toll	PEN (Peruvian currency)	Non available
Parking		Non available

The initial dataset was composed by 8,186 observations; this is individuals who revealed attributes of their actual alternatives modes of transport. Thus, by following the manner in some studies (Fosgerau et.al, 2007), the data needed to be cleaned with respect some factors, these are:

- Unrealistic travel times and costs (i.e. extreme values).
- Unreported or missing values (i.e. in some cases values were not filled for some attributes or income variable).
- Respondents who reported values for just one alternative (since in order to model a discrete choice at least two alternatives are needed).
- Respondents with age less than 6-year-old (i.e. non-decision-makers individuals).
- Error value (e.g. N#A-error).
- The effect of shared-car was excluded from the taxi alternative since this mode of transport is most similar to a public transport rather than taxi.

Once implementing the criterion above, the remaining dataset was composed by the following number of observations:

Table 4-4

Purpose	Number of observations
Commuting	2,443
Business	183
Education	822
Private (e.g. leisure, shopping)	988
Total	4,436

As seen in the table above, the predominant component in the dataset is those individuals who commute, following for those who travel for private purposes. This is consistent with what has been expected since the sample is taken from the basis of the first trip of the day. With respect to the income variable, this was surveyed in eight

categorical levels; in order to incorporate the variable into the models, income has been entered as mid-point following some literature (e.g. Hess et.al, 2008) that introduced this manner. Table 4-5 shows this categorisation:

Table 4-5

Range of income (in PEN)	Category	Mid-point (in PEN)
<600	1	300
601 – 1,000	2	800
1,001 – 1,500	3	1,250
1,501 – 2,000	4	1,750
2,001 -3,000	5	2,500
3,001 – 4,000	6	3,500
4,001 – 7,000	7	5,500
>7,000	8	10,000

It is to remark that the ‘distance to station’ and ‘interchanges’ attributes are disregarded for the purpose of this study –which may encourage further researches including this attributes–. It is also to mention that given the nature of the RP survey the individuals revealed their actual alternatives modes of transport; for this reason some individuals present two rather than three alternatives modes –in particular those that do not own a private car–, nevertheless this effect has properly been computed in the models. This can be illustrated in the following table:

Table 4-6

Income mid-point	<i>Av_{car}</i>		<i>Av_{taxi}</i>		<i>Av_{bus}</i>		Total
	#	%	#	%	#	%	
300	12	1.4%	828	100.0%	828	100.0%	828
800	66	4.2%	1562	100.0%	1,556	99.6%	1,562
1,250	80	8.6%	927	100.0%	923	99.6%	927
1,750	64	13.1%	487	100.0%	481	98.8%	487
2,500	89	28.0%	318	100.0%	314	98.7%	318
3,500	55	28.2%	195	100.0%	195	100.0%	195
5,500	47	52.8%	89	100.0%	89	100.0%	89
10,000	18	60.0%	30	100.0%	30	100.0%	30
Total	431		4,436		4,416		4,436

Therefore by these premises as to the variables surveyed, the following step consists in performing several discrete choice models with this dataset so as to do an empirical work on the detection of income effects and whether or not the econometric specification of income affects the welfare measurement.

4.3.3 Testing on the identification of income effects

4.3.3.1 Residual linear income tier

Having examined the theoretical framework and described concisely the factual description of the case study. It corresponds to test empirically some premises postulated by distinct literature or researches. Thus, all models have been tested in a **multinomial logit** (MNL) form and estimated using BIOGEME (Bierlaire, 2003, 2005). As seen in the literature review section, the additive in income RUM or AIRUM postulated by McFadden –which yield similar statistical results as the basic MNL– serves to prove the presence of non-income effect in the model due to the ‘translation invariance property’. One first group of models therefore has been tested on the basis residual income specification for the overall dataset and for the range of income mid-point categories. This is:

Table 4-7

Estimates	Basic MNL	AIRUM								
		Overall	300	800	1,250	1,750	2,500	3,500	5,500	10,000
ASC_{car}	2.83	2.83	1.21	2.45	2.76	2.92	3.63	2.67	2.95	4.47
(t – test)	11.82	11.82	1.35	4.37	5.54	4.08	5.01	3.34	3.14	2.73
ASC_{taxi}	-2.49	-2.49	-2.58	-2.91	-2.55	-2.27	-1.71	-1.94	-1.24	0.917
(t – test)	-24.26	-24.26	-6.3	-16.6	-9.65	-5.57	-6.08	-4.86	-1.9	0.62
β_{time}	-0.0168	-0.0168	-0.021	-0.0279	-0.0153	-0.00052	-0.0137	-0.0376	-0.051	-0.0349
(t – test)	-4.3	-4.3	-2.25	-4.74	-1.4	-0.03	-1.27	-2.71	-1.8	-0.47
β_{cost}	-0.0637	-	-	-	-	-	-	-	-	-
(t – test)	6.16	-	-	-	-	-	-	-	-	-
λ	-	0.0637	0.104	0.0941	0.0889	0.0256	0.0461	0.0699	0.0548	0.214
(t – test)	-	6.16	1.48	4.05	2.51	1.78	1.6	2.47	1.26	1.1
Individuals	4,436	4,436	828	1562	927	487	318	195	89	30
Parameters	4	4	4	4	4	4	4	4	4	4
Initial LL	-3,241.45	-3,241.45	-578.791	-1,107.02	-673.363	-361.08	-254.885	-157.464	-80.747	-28.093
Final LL	-1,177.04	-1,177.04	-191.969	-331.437	-215.146	-137.495	-111.641	-76.782	-35.617	-12.218
Adj ρ – square	0.636	0.636	0.661	0.697	0.675	0.608	0.546	0.487	-0.509	0.423

Notes: the coloured red numbers denotes the statistical insignificant of the result.

As seen in the table above, all the estimates in the overall model in both specifications –basic and residual income– yields expected results in terms of estimated parameters’ sign (i.e. negative β_{cost} and β_{time} and positive λ) and are statistically significant i.e. the null hypothesis –estimated parameter equals zero– can be rejected. Although, on the other hand, the estimated parameters for all the categories’ income models are consistent with the theory regarding the expected

sign, some of their estimated parameters lack of statistical significance. This latter may be due to the sample size of each group or, in particular, the restricted availability of alternatives that might be not sufficient for the trade-off between the attributes of the alternatives.

Thus, in the attempt of grouping individuals by their income levels (i.e. assuming that there exist statistical homogenous individuals grouped by income that follow some similar pattern of trade-off for the alternatives and sample given), some additional models have been carried out so as to identify the statistical significance of the income classification of individuals. These models are summarised in:

Table 4-8

Estimates	AIRUM overall	Income at three levels			Income at two levels	
		300-800	1,250 -2,500	3,500-10,000	300-800	1,250-10,000
ASC_{car}	2.83	2.14	2.96	3.08	2.14	3.07
(t – test)	11.82	4.54	8.99	5.57	4.54	10.91
ASC_{taxi}	-2.49	-2.83	-2.36	-1.38	-2.83	-2.15
(t – test)	-24.26	-17.41	-14.26	-4.64	-17.41	-15.56
β_{time}	-0.0168	-0.0253	-0.0102	-0.0332	-0.0253	-0.0131
(t – test)	-4.3	-4.97	-1.4	-2.78	-4.97	-2.27
λ	0.0637	0.0904	0.0468	0.0741	0.0904	0.0539
(t – test)	6.16	3.88	4.1	3.22	3.88	5.23
Individuals	4,436	2,390	1,732	314	2,390	2,046
Parameters	4	4	4	4	4	4
Initial LL	-3,241.45	-1,685.82	-1,289.33	-266.304	-1,685.82	-1,555.63
Final LL	-1,177.04	-525.71	-477.011	-133.137	-525.71	-632.522
Adj ρ – square	0.636	0.686	0.626	0.5	0.686	0.591

As seen in Table 4-8 two classifications have been assumed, one first classification assumes the sample of individuals can be subcategorised into three groups of income. The results of this classification shows consistency with respect the expected sign and almost most of the estimates are significant, though the β_{time} estimate for the range of 1,250-2,500 income level is not. On the other hand, the second classification that regards two groups of income level exhibits that models are significant in all estimated parameters and sing-consistency in relation to the theory.

Some important features may be noted across the two classifications. The sample of individuals decreases as the range of income increases; this is most notorious in the

first classification wherein income is at three ranges, this is line with the demographic characteristics of the Peruvian population when is classified by income, i.e. highest income-levels concentrate less relative importance in number of population, (e.g. Chacaltana, 2006; Portugal, 2007). By simple derivation, given the functional form (i.e. residual income), the marginal utility of income, denoted by λ term, shows a decreasing pattern as the range of income increases (this might be interpreted as an increase in income generates a highest increase in utility for those of low-income level). However, this pattern solely observed when classification is at two level of income, at three income-level λ is not continuously decreasing, yet an estimate into the models is not statistical significant.

From the values of the alternative specific constants estimated, it can be inferred that there is a statistical predominance for choosing private car among the individuals. It is to highlight that ASC_{car} increases in accordance with the higher range of income, i.e. the individuals in the sample tend to opt for private car rather than bus; this latter is relative high in the group of 3,500-10,000 range of income in comparison to the 200-800 income-level. Conversely, ASC_{taxi} indicates that the taxi mode of transport is relative less preferred than bus, and decrease as income-levels increases, i.e. individuals with highest income-level opt more for taxi (rather than bus) than those individuals with lower income-level. It is to mention that the relative predominance of car mode may potentially be biased due to its availability; this is most of the respondents that own a car chose this mode rather taxi or bus.

In overall the results yield that statistically the individuals sampled can be classified into two groups without any insignificant estimates in the models tested (in a different study field, Portugal (2007) arrived at similar conclusion). Likewise, the inferences across income-levels obtained from these models are reasonably in accord with what may be expected from reality. Thus these two classifications are to be employed in the following subsection where a more robust testing is made on the detection of income effect.

4.3.3.2 On the manner of Jara-Diaz and Videla

In 1989, Jara-Diaz and Videla researched the presence of income effect in a discrete choice framework applied to commuting in a middle-income corridor of Santiago, Chile. Their empirical work was motivated, among others causes, due to the empirical

specification of income in discrete choice models that implicitly neglected any effect, whereas qualitative analysis suggested the presence of it in developing countries. Thus following the methodology –based on underlying microeconomics framework– provided by the authors, some models have been tested so as to identify whether income effect may be presence. By applying the specification developed in Jara-Diaz and Videla, the following results are obtained:

Table 4-9

Estimates	Basic MML	JD&V
ASC_{car}	2.83	3.65
(<i>t</i> – <i>test</i>)	11.82	13.01
ASC_{taxi}	-2.49	-1.89
(<i>t</i> – <i>test</i>)	-24.26	-14.31
β_{time}	-0.0168	-0.0237
(<i>t</i> – <i>test</i>)	-4.3	-6.07
β_{cost}	-0.0637	-0.238
(<i>t</i> – <i>test</i>)	6.16	-7.58
β_{cost}^2	-	0.00728
(<i>t</i> – <i>test</i>)	-	6.00
Individuals	4,436	4,436
Parameters	4	5
Initial LL	-3,241.45	-3,241.45
Final LL	-1,177.04	-1,152.35
Adj ρ – square	0.636	0.643

Notes: JD&V Jara-Diaz and Videla (1989)

The Table 4-9 contains the main estimates obtained from the specification model derived in Jara-Diaz and Videla along with a basic MNL model. As observed the results are consistent in sign and also statistically significant. In particular the β_{cost}^2 –which is derived from a second-order expansion– is positive as expected. Having estimated the general model in Jara-Diaz and Videla, further analysis may be realised. For this purpose, the income-level segmentations tested in the previous subsection are taken into account; likewise following the analysis developed in the work’s authors, most of the estimates regarding the marginal utility of income is presented in the following table:

Table 4-10

Estimates	JD&V	Income at three levels			Income at two levels	
		300-800	1,250-2,500	3,500-10,000	300-800	1,250-10,000
ASC_{car}	3.65	2.84	1.86	2.06	2.84	1.81

$(t - test)$	13.01	6.17	7.55	3.97	6.17	9.44
ASC_{taxi}	-1.89	-2.2	-2.7	-2.2	-2.2	-2.46
$(t - test)$	-14.31	-11.39	-12.51	-4.52	-11.39	-14.11
β_{time}	-0.0237	-0.0312	-0.0259	-0.064	-0.0312	-0.0276
$(t - test)$	-6.07	-6.44	-4.14	-3.7	-6.44	-5.01
β_{cost}	-0.238	-0.279	-0.0794	-0.0876	-0.279	-0.0772
$(t - test)$	-7.58	-6.00	-2.13	-1.16	-6.00	-2.62
β_{cost}^2	0.00728	0.0096	0.00231	0.00230	0.0096	0.00223
$(t - test)$	6.00	4.86	1.81	0.71	4.86	2.15
$\bar{y}_{average\ weighted}$	1,302.07	337.69	632.55	331.83	337.69	964.38
$\bar{C}1\ (average)$	11.754	11.75	11.75	11.75	11.75	11.75
$\bar{C}2\ (average)$	7.537	7.54	7.54	7.54	7.54	7.54
$\bar{C}3\ (average)$	1.067	1.07	1.07	1.07	1.07	1.07
<i>Diminishing effect</i> ($\beta_{cost}^2 \cdot \bar{y}$)	9.48	3.24	1.46	0.76	3.24	2.15
λ_{c1}	0.15	0.17	0.05	0.06	0.17	0.05
λ_{c2}	0.18	0.21	0.06	0.07	0.21	0.06
λ_{c3}	0.23	0.27	0.08	0.09	0.27	0.07
$\bar{\lambda}_{weighted\ by\ market\ share}$	0.22	0.26	0.07	0.08	0.26	0.07
$\bar{\lambda} \cdot \bar{y}$	287.42	86.53	46.76	27.26	86.53	69.35
<i>Individuals</i>	4,436	2,390	1,732	314	2,390	2,046
<i>Parameters</i>	5	5	5	5	5	5
<i>Initial LL</i>	-3,241.45	-1,685.82	-1,289.33	-266.304	-1,685.82	-1,555.63
<i>Final LL</i>	-1,152.35	-514.654	-483.024	-135.861	-514.654	-642.108
<i>Adj ρ – square</i>	0.643	0.692	0.621	0.471	0.692	0.584

Although all the results, as observed in Table 4-10, are consistent in sign coefficient, the estimates of β_{cost} in 3,500-10,000 income-level model is not statistically significant nor the β_{cost}^2 in the last two models of the first classification. Conversely, the estimates of the models in the second classification are robust in sign and significance. The underlying hypothesis of β_{cost} and β_{cost}^2 is to be decreasing with high ranges of income; the results indicates that the estimated parameters satisfy the hypothesis, excepting β_{cost} in third model of the first classification compared with its previous pair, nor the estimates are insignificant; as opposed the models of the second classification accomplished accurately the expected premises with respect the pattern of the estimates across incomes, i.e. both estimates are decreasing with income.

The marginal utility of income is denoted by the λ term and derived from the equation (16) in Jara-Diaz and Videla (1989); this is $-\beta_{cost} - \beta_{cost}^2 \cdot C_j$. Thus it may be seen that λ is properly decreasing across and overall alternatives of the models within the second classification, though this do not occur between the two last models of the first classification³⁷. For instance, the estimated $\bar{\lambda}$ into the second classification diminishes from 0.26 to 0.07; likewise the pattern of $\bar{\lambda} \cdot \bar{y}$ is decreasing, they pass from 86.53 to 69.25 for the ranges of 300-800 and 1,250-10,000; respectively.

Thus far it is to note that the properties –i.e. the coefficient of β_{cost} , β_{cost}^2 and $\bar{\lambda}$ and their values in $\beta_{cost} \cdot \bar{y}$ and $\bar{\lambda} \cdot \bar{y}$ are diminishing– postulated in the work of Jara-Diaz and Videla are accurately accomplished for the second classification. Nevertheless, a final testing ought to be done in order to determine whether the estimates in $\bar{\lambda}$ between the groups of income-level are statistical significance. Thus by following the test proposed by the authors in their equation (22), we obtain:

Table 4-11

Income groups	Difference in $\bar{\lambda}_j$	\bar{t}
$(Y_{300-800})(Y_{1,250-2,500})$	0.182	3.300
$(Y_{300-800})(Y_{3,500-10,000})$	0.174	2.120
$(Y_{300-800})(Y_{1,250-10,000})$	0.184	3.618

The t-test calculated reflects the statistical significance of the difference in marginal utility of income ($\bar{\lambda}$) between the models³⁸ for the sample given; i.e. the broadly considered assumption of constancy in marginal utility of income could not be any longer hold for estimations on the basis of the dataset carried out in this study. Thus, having statistically detected –on the basis of Jara-Diaz and Videla– the presence of income effects due to the non-constancy of the marginal utility of income across of individuals at different levels of income, a wide analysis can be implemented by testing distinct non-linear income specification forms so as to identify some possible implications of income effects.

³⁷ This latter may be explained as a consequence of less relative weight for the sample of individuals with income between 3,500 and 10,000.

³⁸ Even when some estimates in the models of the first classification are insignificant, their difference in λ is not.

4.3.3.3 Functional forms for the treatment of income effects

In Batley and Ibanez (2013), one of the aim of their work is to outline conditions under the Marshallian demand is unaffected by non-linear income expansion path. The authors related these conditions to some well-known practical model specifications, as seen in Table 3-1. Some of these specifications are in compliance with the path independence property with regards income, prices or both. Thus far, most of the models have been carried out by assuming the 'residual income' specification form developed in McFadden (1981) –'model A' in Batley and Ibanez (2013)– which guaranties path independence in income and prices –any income effect is capture by the numeraire good–. In order to empirically identify the implications of specifications form in discrete models with respect the path independence property, we are considering the models from A to F in the table 1 of Batley and Ibanez (2013) –also illustrated in Table 3-1 of the present document–. It ought to be mentioned that in the attempt to obtain significance in the estimated parameters, the upper and the lower bounds on the values of the coefficients for some models have been extended following the Biogeme instructions for practical cases (Biogeme Turtorial, 2008); though some models have been carried out in a narrower range of possible values (from -10 to 10). The models also have been tested in their restricted and unrestricted form.

Table 4-12

Estimates	MNL	B&I A		B&I B		B&I C		B&I D		B&I E		B&I F	
		$\tilde{v}_j = \lambda(y - p_i) + \varepsilon_j$	$\tilde{v}_j = \beta_j(y^\alpha - p_i) + \varepsilon_j$	Unrest.	Rest.	Unrest.	Rest.	Unrest.	Rest.	Unrest.	Rest.	Unrest.	Rest.
ASC_{car}	2.83	2.83	2.08	2.83	4.43	4.95	2.51	2.53	1.98	2.15	51.5	5.04	
(<i>t</i> - test)	11.82	11.82	8.18	11.82	9.33	14.15	11.12	11.18	4.16	11.36	2.76	14.43	
ASC_{taxi}	-2.49	-2.49	-2.50	-2.49	-0.861	-0.637	-2.59	-2.46	-3.23	-2.72	31.2	-0.111	
(<i>t</i> - test)	-24.26	-24.26	-14.53	-24.26	-3.15	-3.21	-18.00	-22.18	-14.68	-25.66	3.47	-0.46	
β_{time}	-0.0168	-0.0168	-0.0154	-0.0168	-0.0147	-0.0166	-0.018	-0.0183	-0.019	-0.00951	-0.0173	-0.0941	
(<i>t</i> - test)	-4.3	-4.3	-3.25	-4.3	-3.15	-4.58	-4.79	-4.96	-4.78	-2.37	-4.63	-1.57	
β_{cost}	-0.0637	-	-	0.0637	-	-	-	-59.5	-	7.6E-11	-	83.30	
(<i>t</i> - test)	6.16	-	-	6.16	-	-	-	-4.71	-	6.05	-	1.56	
β_{cost_1}	-	-	0.0489	-	-	-	-82.5	-	0.19	-	-0.001	-	
(<i>t</i> - test)	-	-	4.42	-	-	-	-6.91	-	0.45	-	-222.44	-	
β_{cost_2}	-	-	0.127	-	-	-	-92.4	-	0.19	-	-0.001	-	
(<i>t</i> - test)	-	-	3.98	-	-	-	-4.04	-	0.45	-	-218.87	-	
β_{cost_3}	-	-	0.694	-	-	-	361	-	0.19	-	-0.001	-	
(<i>t</i> - test)	-	-	4.04	-	-	-	-5.43	-	0.45	-	-235.77	-	
λ	-	0.0637	-	-	-	15.00	-	-	-	-	-	-	
(<i>t</i> - test)	-	6.16	-	-	-	1.00	-	-	-	-	-	-	
λ_1	-	-	-	-	-14.1	-	-	-	-	-	-	-	
(<i>t</i> - test)	-	-	-	-	-76.65	-	-	-	-	-	-	-	
λ_2	-	-	-	-	-14.1	-	-	-	-	-	-	-	
(<i>t</i> - test)	-	-	-	-	-76.65	-	-	-	-	-	-	-	
λ_3	-	-	-	-	-14.1	-	-	-	-	-	-	-	
(<i>t</i> - test)	-	-	-	-	-76.65	-	-	-	-	-	-	-	
α	-	-	-0.136	-1.92E-15	-0.112	0.0773	-	-	0.861	7.6E-11	-0.0184	-0.255	
(<i>t</i> - test)	-	-	-74.91	1.00	-9.55	12.28	-	-	3.3	0	-5.5	-6.68	
γ	-	-	-	-	-	-	-	-	-	-	0.00016	-0.0941	
(<i>t</i> - test)	-	-	-	-	-	-	-	-	-	-	10.57	-1.57	
Individuals	4,436	4,436	4,436	4,436	4,436	4,436	4,436	4,436	4,436	4,436	4,436	4,436	
Parameters	4	4	7	5	7	5	6	4	7	5	8	6	
Initial LL	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.447	-3,241.4	-3,241.447	
Final LL	-1,177.036	-1,177.036	-1,157.991	-1,177.036	-1,087.203	-1,135.384	-1,149.731	-1,164.004	-1,127.644	-1,197.633	-1,100.00	-1,100.808	
Adj ρ - square	0.636	0.636	0.641	0.635	0.662	0.648	0.643	0.64	0.65	0.629	0.658	0.659	

Table 4-12 compiles the main estimated results from the distinct specifications form outlined in Batley and Ibanez (2013). By a first glance it may be observed that two models exhibit insignificance in some of their estimated parameters, but at least one model of each specification form (either the unrestricted or restricted one) is statistically significant in all their estimates. The MNL and ‘B&I A’ models that have been run in previous subsections do not yield unexpected results (see further analysis in 4.3.3.1 and 4.3.3.2). Thus let focus on the other specifications form; either the unrestricted and restricted ‘B&I B’ model present statistical significance in their estimates. It may be noted that the magnitude of the estimates in the restricted model are similar than their pairs in the MNL and ‘residual income’ specification form. A simple numerical calculation of $\partial \tilde{v}_j / \partial y$ for both models suggests that although the values do not coincide in sign, both converges to zero. In a similar manner when calculating the effect from a lump-sum of income; this is $\partial(\partial \tilde{v}_j / \partial y) / \partial y$, the value is very close to zero which is in line with the path independence on income property under the ‘case 2’ postulated in Batley and Ibanez (2013).

Both the unrestricted and restricted ‘B&I C’ and ‘B&I D’ models –whereas the first one guaranties path independence on income and prices, the second not– yields consistent estimates in line with the underlying theory. In particular the path independence property on income does not hold for the case of ‘B&I D’; indeed the magnitude of the $\partial(\partial \tilde{v}_j / \partial y) / \partial y$ is really close to zero and negatively increasing as larger income changes. In the case of ‘B&I E’ and ‘B&I F’ models which exhibit path dependence in prices and incomes yields significant estimates for the restricted and unrestricted models, respectively.

In overall, it may be inferred that those models that hold the path independence property present approximate results with respect to the basic MNL or ‘residual income’ models. It may be said that the aggregation across goods based on these models –MNL, ‘residual income’, ‘B&I B’ and ‘B&I C’– should not be statistically different meaning that the differences in their estimates should statistically equal to zero, in contrast with path dependence models.

Having estimated models with different specifications form, it turns to test them in hypothetical scenarios, thereby identifying possible implications in their application in welfare measurement; as seen in the following subsection.

4.3.4 Simulation and sensitive analysis in the log-sum

By using the previous models in their distinct specifications, some simulation in the estimated parameters of time and cost have been performed. Then by obtaining new estimates, these have been introduced to the model so as to develop two states scenarios. These hypothetical scenarios has been utilised to represent the log-sum as a potential measure of welfare. To illustrate some potential changes in welfare as a result of the introduction of the new metro system, the estimated parameters of the alternative 'bus' is regarded as adequate proxy for this purpose, this is because both services belong to the public transport sphere and it is reasonable expected a significant substitution between these modes. Thus three variations have been tested into the models; the first regards solely an increase in the public service transport (bus), the second introduces a decrease in travel time in the public transport and the final is a simultaneous combination of two first. These can be seen in the following table:

Table 4-13

Estimates	$\tilde{v}^0(p^0, t^0)$	$\tilde{v}^1(p^1, t^0)$	$\tilde{v}^2(p^0, t^1)$	$\tilde{v}^3(p^1, t^1)$	$\tilde{v}^0(p^0, t^0)$	$\tilde{v}^1(p^1, t^0)$	$\tilde{v}^2(p^0, t^1)$	$\tilde{v}^3(p^1, t^1)$
	($\Delta = 0$)	($\Delta p = 0.5$)	($\Delta t = -0.4$)	($\Delta p; \Delta t$)	($\Delta = 0$)	($\Delta p = 0.5$)	($\Delta t = -0.4$)	($\Delta p; \Delta t$)
	MNL				B&I A			
$\ln \sum_{j=1} e^{\tilde{v}_{nj}}$	8.52979	8.38732	8.567941	8.57821	91.47190	102.65747	155.93925	121.20763
p^i	1	1.5	1	1.500	1	1.5	0.6	0.6
t^i	1	1	0.6	0.600	1	1	1	1.5
π_{car}	8.83%	8.93%	8.66%	8.68%	8.88%	8.93%	9.40%	8.68%
π_{taxi}	5.58%	6.97%	5.11%	5.31%	6.81%	6.97%	18.51%	5.31%
π_{bus}	85.60%	84.10%	86.23%	86.00%	84.31%	84.10%	72.09%	86.00%
<i>Fianl LL</i>	-1,177.036	-1,176.29	-981.421	-1,005.738	-1,177.036	-1,176.29	-1,945.203	-1,005.738
<i>Adj. ρ – square</i>	0.636	0.637	0.696	0.688	0.636	0.636	0.399	0.688
	B&I B				B&I D			
$\ln \sum_{j=1} e^{\tilde{v}_{nj}}$	8.09234	8.87932	8.40059	8.02529	8.51057	8.03950	8.47641	8.59223
p^i	1	1.5	1	1.5	1	1.5	1	1.5
t^i	1	1	0.6	0.6	1	1	0.6	0.6
π_{car}	8.87%	9.18%	8.83%	9.06%	8.24%	8.20%	7.81%	7.75%
π_{taxi}	6.88%	9.41%	5.21%	7.47%	4.42%	4.10%	3.14%	3.04%
π_{bus}	84.25%	81.41%	85.96%	83.47%	87.34%	87.70%	89.05%	89.20%
<i>Fianl LL</i>	-1,157.991	-1,367.299	-1,367.299	-1,183.623	-1,149.731	-831.303	-751.536	-682.058
<i>Adj. ρ – square</i>	0.641	0.576	0.697	0.633	0.643	0.742	0.766	0.788
	B&I E				B&I F			
$\ln \sum_{j=1} e^{\tilde{v}_{nj}}$	8.72233	8.70863	8.94338	8.89484	21.22041	-4.43097	4.49719	5.155731
p^i	1	1.5	1	1.5	1	1.5	1	1.5
t^i	1	1	0.6	0.6	1	1	0.6	0.6
π_{car}	8.88%	9.09%	8.90%	8.83%	9.56%	9.67%	9.14%	9.38%
π_{taxi}	6.81%	6.80%	5.51%	5.51%	37.12%	45.57%	12.89%	23.54%
π_{bus}	84.31%	84.11%	85.59%	85.66%	53.32%	44.76%	77.97%	67.08%
<i>Fianl LL</i>	-1,197.633	-1,171.184	-1,045.719	-1,045.719	-1,100.808	-2,354.693	-1,574.372	-1,907.156
<i>Adj. ρ – square</i>	0.629	0.637	0.676	0.676	0.659	0.272	0.512	0.41

The differences between $\ln \sum_{j=1} e^{\tilde{v}_{nj}}$ correspond to the measure of the logsum. For instance, in the basic MNL when prices are assumed to increase the new utility stated is lower than before the change. As opposed, the decreases in time attribute in 40% yields a lower stated than without change. By comparing across models, it may be reasonable expected similar results in the MNL basic and the ‘residual income’ (B&I A) model; nonetheless, this does not occur. Although the models hold the path independence property and their estimated parameters are significant, they yield unexpected results. It may be noted that when the models were initially estimated

their estimates were rather similar, however, once they are simulated by a sensitive testing of their estimates, the models tend to differ in the size of the outcomes. Thus, this latter may exemplify that even when two specifications form theoretically hold equal properties, these might differ in outcomes when run. However, this instance is not sufficient to arrive to that conclusion; indeed the dataset also may be playing a role in this divergence.

In addition, the other models presented in the table above exhibit some insignificance in their estimates. Despite of it, those models serve to observe the existence of some patterns in the logsum. Some of them such as B&I B, D and E present the expected pattern between price and disutility or reduce in time and utility. Nevertheless, when effects are combined the overall results vary across these models. On the other hand, the Cobb-Douglas functional form (B&I F) yields rather different market shares (expressed in π_j) compared with those pairs.

5. CONCLUSIONS AND RECOMMENDATIONS

From the microeconomic foundations two approaches may be distinguished, the preference-based and the choice-based approach. Both comply with the rationale axioms of the individual decision-making. The principles of these approaches are supported within the neoclassical theory of consumer behaviour and the Random Utility Maximisation Models, respectively. Thus, the rational economic behaviour can be researched from the basis of them; indeed these two branches have developed their theory and empirical applications by the use of different mathematical and statistical tools. In the attempt to converge these two approaches, many researches have been realised. The classical work of Small and Rosen (1981) attempts, among other objectives, to fill the gap of these two approaches in the field of welfare measure. The tractability feature of discrete choice models have provoked that the theoretical principles of welfare analysis might be applied in its framework.

Even though the framework of discrete choice models may enable the computation of welfare measurement, there are still some conditions that are necessarily to be hold. When aggregation across commodities is to calculate, the path-dependency problem may arise; however, the literature has addressed this issue by introducing the numeraire good, thus this problem is overcome. The underlying conception of the

numeraire good stands that the set of commodities under the decision-making of the individual are relative small with respect to their income. In line with the literature, this is rather common in transport sector where the fare or cost of transport is proportionally small in the budget of the individual. The formal expression of the numeraire good is derived from the quasi-linear utility function. A relevant contribution on the path-independency field, Batley and Ibanez (2010) postulates the assumptions that need to be hold, or implicitly assumed in the work of Small and Rosen, for the path-independent property in discrete-continuous demand and probabilistic demand in the framework of discrete choice.

When aggregation is across individuals (strong) assumptions are to be introduced, in this regard even in a discrete choice framework the constancy of marginal utility of income is to be assumed. The importance of aggregate across individuals rests on making welfare analysis. As mentioned, the work of Small and Rosen attempts to harmonise the two individual's decision-making approaches with the intention of postulating a framework for welfare calculation; although the authors ended up assuming implicitly the Marshallian demand which theoretically hold path-dependent problem.

Some other methods have been proposed in the literature to measuring welfare in discrete choice models. The most common one is the logsum, but still not widely applicable in formal appraisal. Methods more sophisticated have been developed in the work of Karlstrom (2000) where the identification of groups of individuals before and after a given change is crucial for accurate calculation; however, this condition seems to be less suitable in practice. Mcfadden, on the other hand, has also postulated a framework for the welfare measurement throughout a simulation process.

On the basis of this theory, the present study has tested several models on the framework of discrete choice by using the multinomial logit models. The dataset collected from an urban transport survey Lima Metropolitan in 2004 has enabled to carry several models. The first set of models has sought to detect the presence of income effects, thus by assuming an income tier it has been found the individuals surveyed in the study can be classified into two representative income-groups. An additional test, by following the Jara-Diaz and Videla (1989), shows that the marginal

utility of income decreases with income and these are statistically different when are calculated among groups. As in the author's work, these results may induce to further research in this field for the case of income effects in developing countries.

By assuming the set of specifications in proposed in Batley and Ibanez (2013), several models have been carried out. It has found the models are sensitive to the specification form; indeed it can be seen that some models might yield substantial different results even when they come from the same dataset. In this sense, since the dataset was constructed from a revealed preference survey, it may be reasonable presume to find higher variations in the values of the attributes which in some extend may be reflected in the models with different specifications. A final testing group is regarding to the measure of logsum, for this purpose variations in the size of the time and cost variables have been introduced so as to replicate hypothetical scenarios that might been observed with the introduction of the metro system. Although most of the models are consistent with the expected pattern between price, time and utility or disutility; some estimates of these models are not statistically significant.

Some limitations have been found in the attempt of measuring welfare, in particular it may be presumed that the models are not performed well within a complex specification. The assumption of mid-points might also influence the results of complex specification; especially when the power term is introduced into the some coefficients. It may be also mentioned that given the nature of the dataset a nested model may be carried out since the taxi alternative may be correlated with the choice of private cars. Finally, the results presented for the Peruvian case of study might cast light on future investigations for the detection of income effects and welfare measurement in the Peruvian transport projects appraisal.

6. ACKNOWLEDGEMENTS

I would like to thank to my supervisor Dr. Richard Batley for letting me be part of this exciting subject, for his guidance and for sharing his invaluable knowledge in this field. I would also like to express my deepest gratitude to my whole family, especially to my parents, Cirilo and Rosa, my sister Maritza and my nephew Nicolas for their

constantly support and encouragement in this year. And finally, I am extremely grateful to Servir for their financing support during this academic year.

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